بسم الله الرحمن الرحیم

Security and Privacy in Wireless Networks

Mohammad Hossein Manshaei

manshaei@gmail.com

A Tutorial on Game Theory for Wireless Networks

Appendix B

Security and Cooperation in Wireless Networks

secowinet.epfl.ch

static games, dynamic games, repeated games, strict and weak dominance, Nash equilibrium, Pareto optimality, and subgame perfection

Chapter outline

B.1 Introduction

B.2 Static games

B.3 Dynamic games

B.4 Repeated games

Brief introduction to Game Theory

- Discipline aiming at modeling situations in which actors have to make decisions which have mutual, **possibly conflicting**, consequences
- Classical applications: **economics**, but also politics and biology
- Example: should a company invest in a new plant, or enter a new market, considering that the **competition** *may* make similar moves?
- Most widespread kind of game: **non-cooperative** (meaning that the players do not attempt to find an agreement about their possible moves)

Classification of games

Non-cooperative	Cooperative
Static	Dynamic (repeated)
Strategic-form	Extensive-form
Perfect information	Imperfect information
Complete information	Incomplete information

Perfect information: each player can observe the action of each other player.

Complete information: each player knows the identity of other players and, for each of them, the payoff resulting of each strategy.

Applications of Game Theory (Summary)

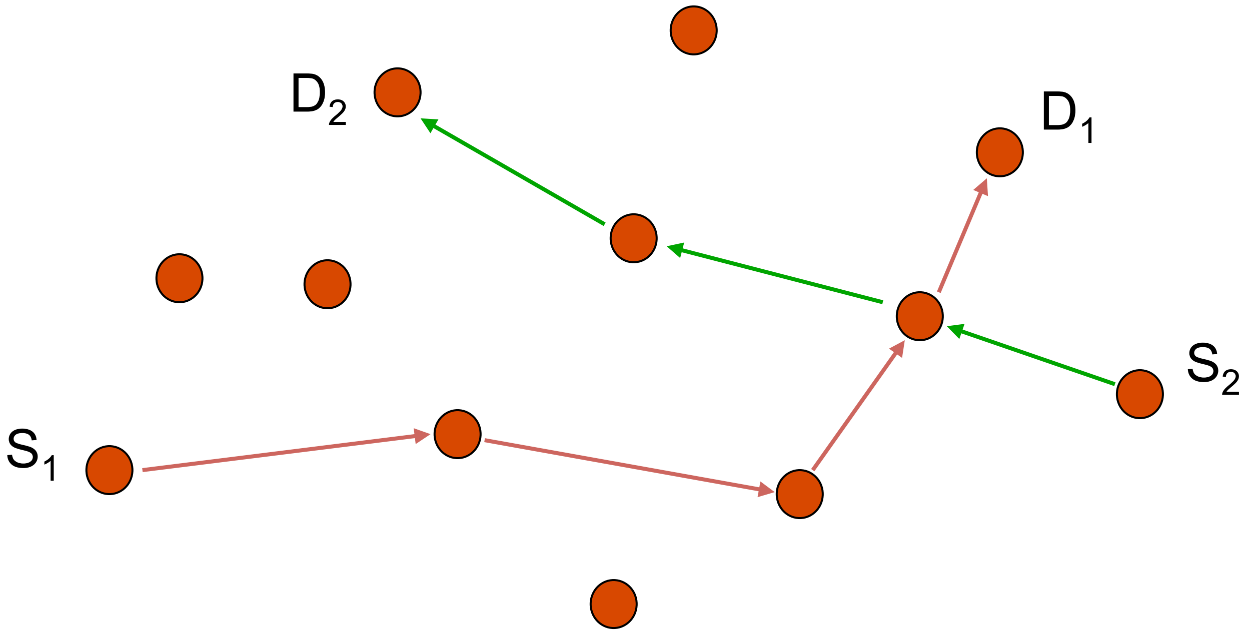
- Recent interest in networked-systems (communication, transportation networks, and electricity markets).
  - Large-scale networks emerged from interconnections of smaller networks and their operation relies on various degrees of **competition** and **cooperation**.
  - Online advertising on the Internet: Sponsored search **auctions**.
  - **Distributed control** of competing heterogeneous users.
  - Information evolution and belief propagation in **social networks**.
  - Sustainability and smart grids.
- "Recently" applied to computer networks
  - Nagle, RFC 970, 1985
    - "datagram networks as a multi-player game"
  - Paper in first volume of IEEE/ACM ToN (1993)
  - Wider interest starting around 2000

Limitations of Game Theory

- No unified solution to general conflict resolution

- Real-world conflicts are complex
  - models can at best capture important aspects
- Players are (usually) considered rational
  - Determine what is best for them given that others are doing the same
- No unique prescription
  - Not clear what players should do
- **But it can provide intuitions, suggestions and partial prescriptions**
  - **best mathematical tool we currently have**

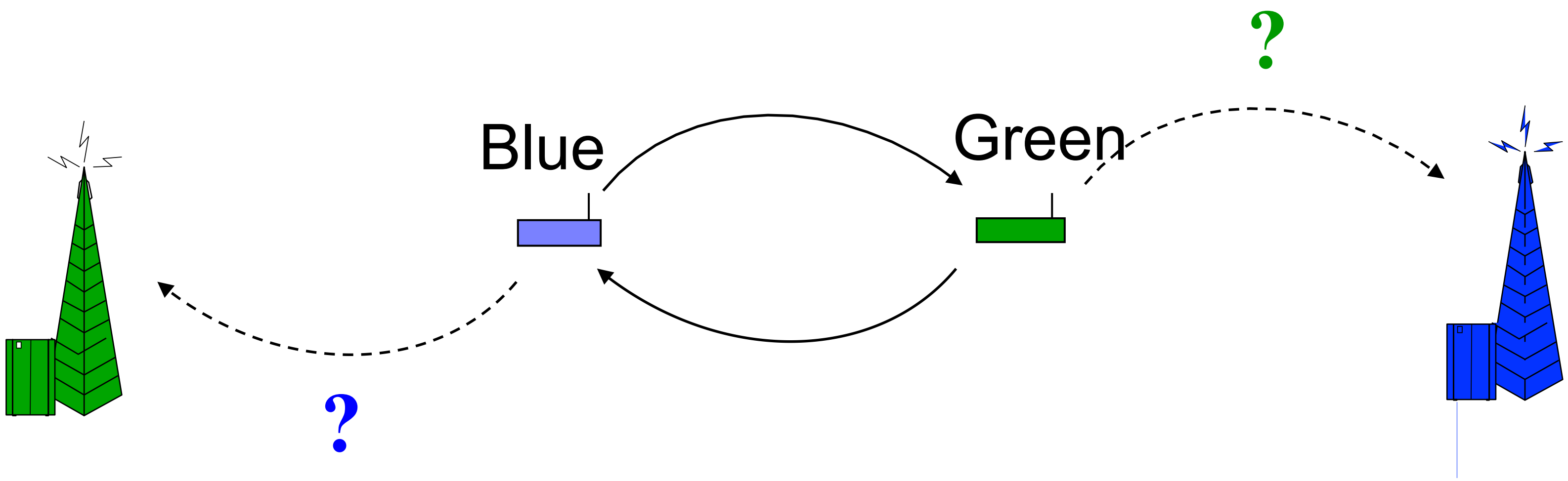
Cooperation in self-organized wireless networks



Usually, the devices are assumed to be cooperative.
But what if they are not?

Chapter outline

Example 1: The Forwarder’s Dilemma



From a problem to a game

- users controlling the devices are ***rational*** = try to maximize their benefit
- game formulation: G = (P,S,U)
  - P: set of players
  - S: set of strategy functions
  - U: set of payoff functions →

• Reward for packet reaching the destination: 1
• Cost of packet forwarding: c (0 < c << 1)

- ***strategic-form*** representation

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 1)
Drop	(1, -c)	(0, 0)

Solving the Forwarder's Dilemma

Strict dominance: strictly best strategy, for any strategy of the other player(s)

Strategy s_i strictly dominates if

$$u_i(s_i^{'}, s_{-i}) < u_i(s_i, s_{-i}), \forall s_{-i} \in S_{-i}, \forall s_i^{'} \in S_i$$

where: $u_i \in U$ payoff function of player *i*

$s_{-i} \in S_{-i}$ strategies of all players except player i

In Example 1, strategy Drop ***strictly dominates*** strategy Forward

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 1)
Drop	(1, -c)	(0, 0)

Solving the Forwarder's Dilemma

Solution by iterative strict dominance:

Blue \ Green	Forward	Drop
Forward	~~(1-c, 1-c)~~	~~(-c, 1)~~
Drop	~~(1, -c)~~	(0, 0)

Drop ***strictly dominates*** Forward

BUT

Forward would result in a ***better outcome***

} Dilemma

Repeated Iterative Strict Dominance

Strict dominance: strictly best strategy, for any strategy of the other player(s)

Blue \ Green	X	Y	V	W
A	(2, 9)	(0, 6)	(0, 4)	(1, 2)
B	(1, 3)	(3, 6)	(6, 1)	(6, 3)
C	(4, 2)	(4, 1)	(2, 2)	(8, 3)
D	(3, 7)	(4, 5)	(2, 6)	(4, 7)

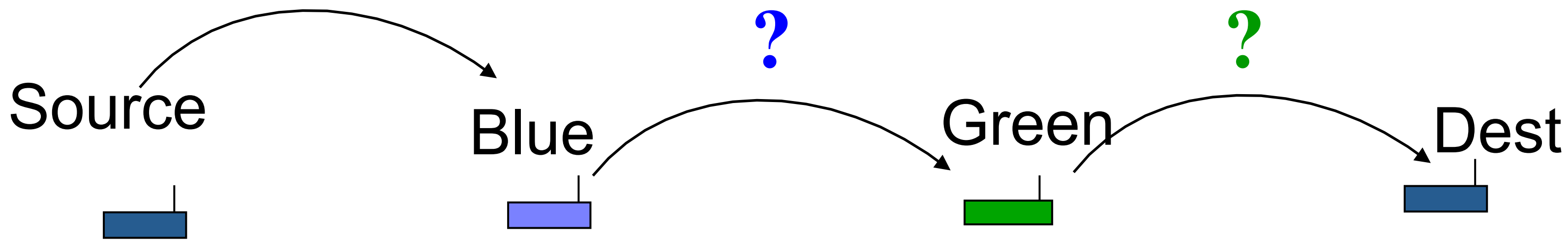
Cold War!

Country 1 \ Country 2	Reduce military investment	Increase military investment
Reduce military investment	(1, 1)	(-1, 2)
Increase military investment	(2, -1)	(0, 0)

Payoffs:

- **2:** I have weaponry superior to the one of the opponent
- **1:** We have equivalent weaponry and managed to reduce it on both sides
- **0:** We have equivalent weaponry and did not managed to reduce it on both sides
- **-1:** My opponent has weaponry that is superior to mine

Example 2: The Joint Packet Forwarding Game



- Reward for packet reaching the destination: 1
- Cost of packet forwarding: c (0 < c << 1)

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 0)
Drop	(0, 0)	(0, 0)

No strictly dominated strategies !

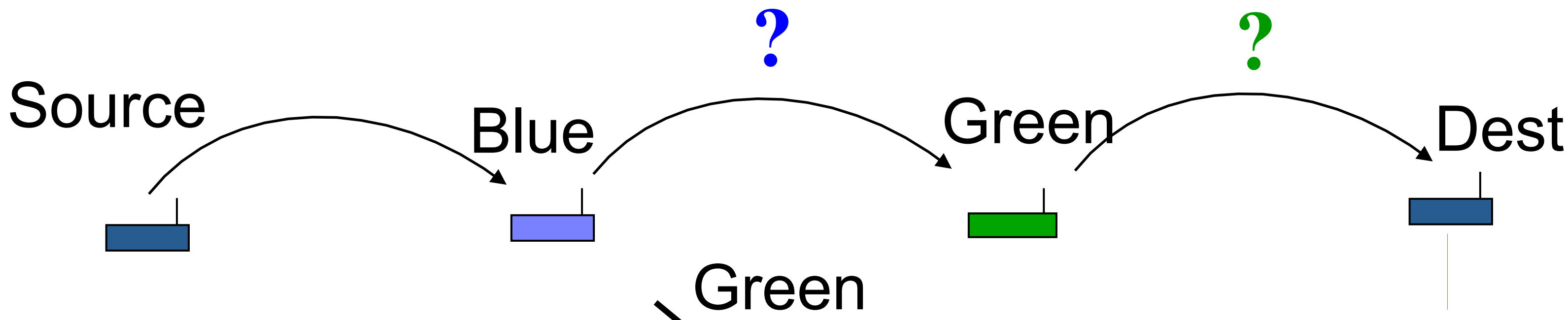
Weak dominance

Weak dominance: strictly better strategy for at least one opponent strategy

Strategy s'_i is weakly dominated by strategy s_i if

$$u_i(s_i', s_{-i}) \le u_i(s_i, s_{-i}), \forall s_{-i} \in S_{-i}$$

with strict inequality for at least one s_{-i}



Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	~~(-c, 0)~~
Drop	~~(0, 0)~~	~~(0, 0)~~

Iterative weak dominance

BUT

The result of the iterative weak dominance is not unique in general !

Repeated Iterative Weak Dominance

Weak dominance: strictly better strategy for at least one opponent strategy

Blue \ Green	X	Y	V	W
A	(3, 9)	(0, 6)	(0, 4)	(1, 2)
B	(1, 3)	(3, 6)	(6, 1)	(6, 3)
C	(4, 2)	(4, 1)	(2, 2)	(8, 2)
D	(3, 7)	(4, 5)	(2, 6)	(4, 7)

Nash equilibrium (1/2)

Nash Equilibrium: no player can increase its payoff by deviating unilaterally

E1: The Forwarder's Dilemma

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 1)
Drop	(1, -c)	(0, 0)

E2: The Joint Packet Forwarding game

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 0)
Drop	(0, 0)	(0, 0)

Nash equilibrium (2/2)

Strategy profile s^* constitutes a **Nash equilibrium** if, for each player *i*,

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in S_i$$

where: $u_i \in U$ payoff function of player i

$s_i \in S_i$ strategy of player i

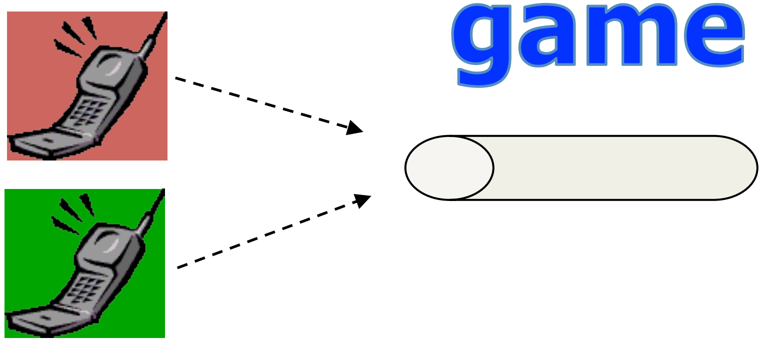
The **best response** of player *i* to the profile of strategies s_{-i} is a strategy s_i such that:

$$b_i(s_{-i}) = \arg\max_{s_i \in S_i} u_i(s_i, s_{-i})$$

Nash Equilibrium = Mutual best responses

Caution! Many games have more than one Nash equilibrium

Example 3: The Multiple Access game



Time-division channel

Reward for successful transmission: 1

Cost of transmission: c
(0 < c << 1)

Blue \ Green	Quiet	Transmit
Quiet	(0, 0)	(0, 1-c)
Transmit	(1-c, 0)	(-c, -c)

There is no strictly dominating strategy

There are two Nash equilibria

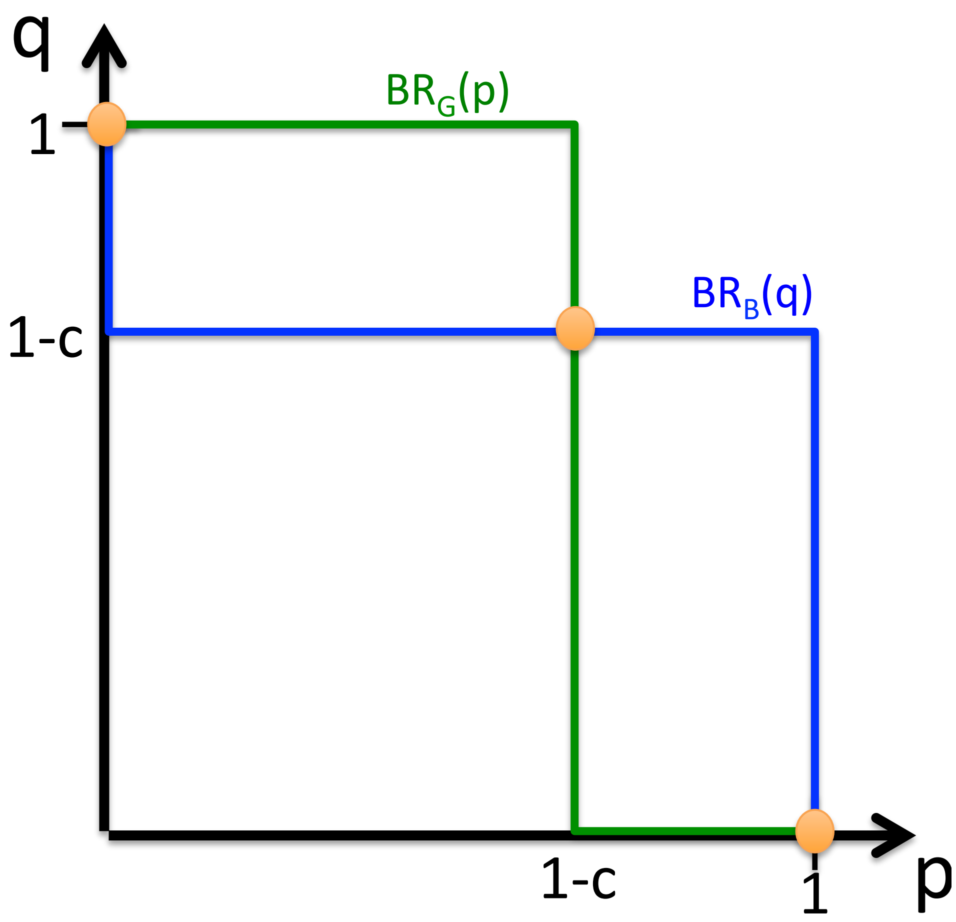
Mixed strategy Nash equilibrium

p: probability of transmit for Blue

q: probability of transmit for Green

$$u_{blue} = p(1-q)(1-c) - pqc = p(1-c-q)$$

$$u_{green} = q(1-c-p)$$



objectives

- Blue: choose p to maximize u_{blue}
- Green: choose q to maximize u_{green}

$$p^* = 1-c, \; q^* = 1-c$$

is a Nash equilibrium

Example 4: The Jamming game

transmitter 

jammer 

two channels: C_1 and C_2

transmitter:
- reward for successful transmission: 1
- loss for jammed transmission: -1

jammer:
- reward for successful jamming: 1
- loss for missed jamming: -1

Blue \ Green	C_1	C_2
C_1	(-1, 1)	(1, -1)
C_2	(1, -1)	(-1, 1)

There is no pure-strategy Nash equilibrium

$p = \frac{1}{2}, \; q = \frac{1}{2}$ is a Nash equilibrium

p: probability of transmit on C_1 for Blue

q: probability of transmit on C_1 for Green

Nash Theorem, 1950

Every Finite Game has a Mixed-strategy Nash-Equilibrium.

Efficiency of Nash equilibria

E2: The Joint Packet Forwarding game

Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 0)
Drop	(0, 0)	(0, 0)

How to choose between several Nash equilibria ?

Pareto-optimality: A strategy profile is Pareto-optimal if it is not possible to increase the payoff of any player without decreasing the payoff of another player.

Efficiency

Pareto-optimality: It is not possible to increase the payoff of any player without decreasing the payoff of another player.

Blue \ Green	X	Y	V	W
A	(3, 9)	(0, 6)	(0, 4)	(1, 2)
B	(1, 3)	(3, 6)	(6, 1)	* (6, 3)
C	(4, 2)	(4, 1)	(2, 2)	* (8, 2)
D	(3, 7)	* (4, 5)	(2, 6)	* (4, 7)

How to study Nash Equilibria ?

Properties of Nash equilibria to investigate:

- uniqueness
- efficiency (Pareto-optimality)
- emergence (dynamic games, agreements)