



# Mobile Networking

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An Analytical Approach: Bianchi Model

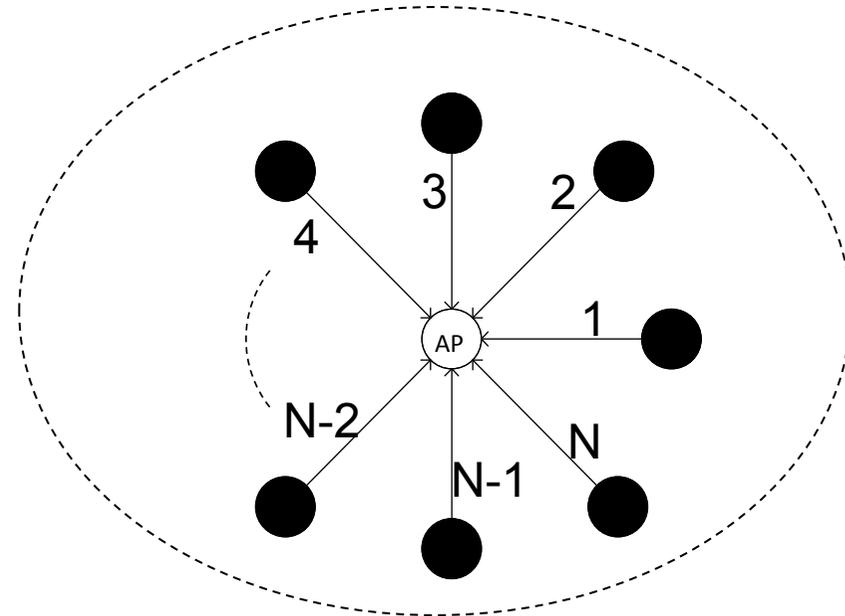
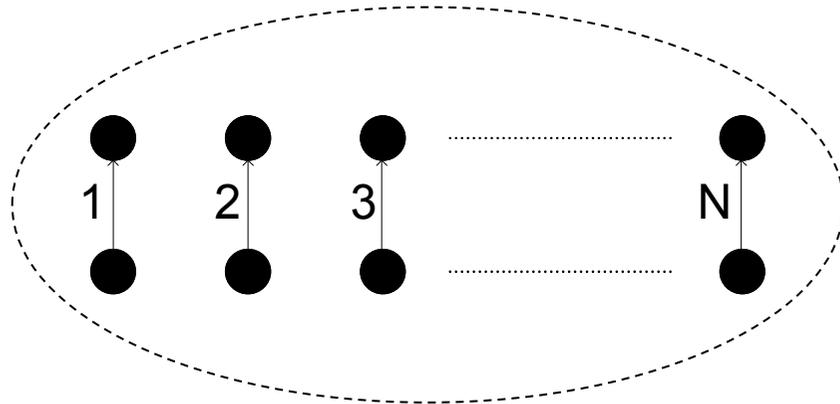
# **WLAN PERFORMANCE ANALYSIS**

# Performance Evaluation of IEEE 802.11 (DCF)

- Real Experimentations
  - HoE on IEEE 802.11b
- Analytical Models
  - Bianchi's Model
- Simulations
  - ns-2

# Bianchi's Model: Topology and Parameters

- N links with the same physical condition (single-collision domain):



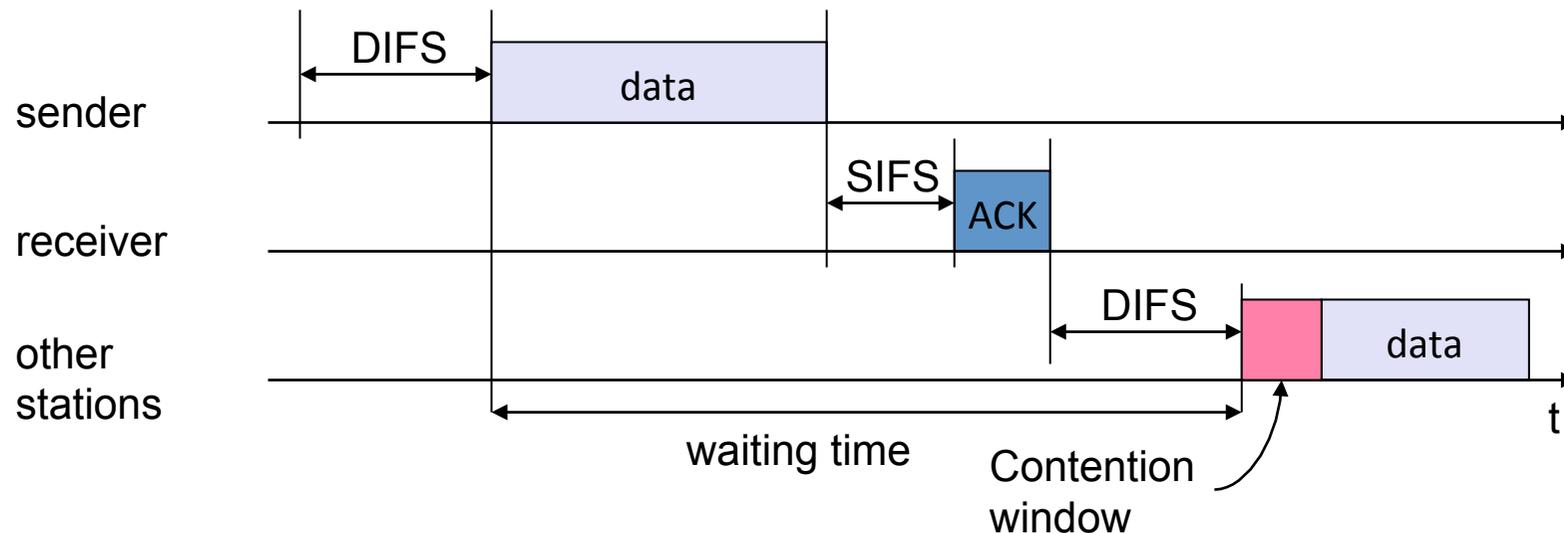
We want to calculate the throughput of this network.

MAC Layer
PHY Layer

$\pi$  = Probability of Transmission  
 $\updownarrow$   
 $p$  = Probability of Collision  
 = More than one transmission at the same time  
 =  $1 - (1 - \pi)^{N-1}$

# 802.11 - CSMA/CA unicast (Review)

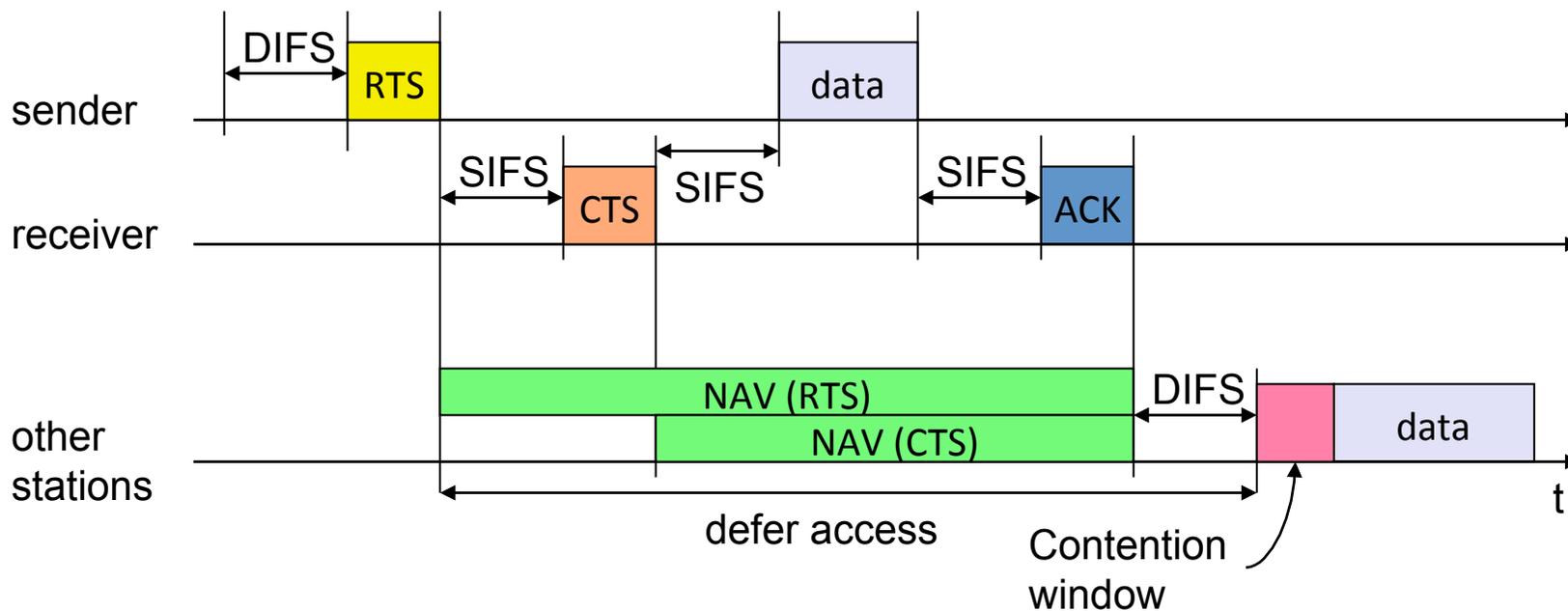
- Sending unicast packets
  - station has to wait for DIFS before sending data
  - receiver acknowledges at once (after waiting for SIFS) if the packet was received correctly (CRC)
  - automatic retransmission of data packets in case of transmission errors



The ACK is sent right at the end of SIFS  
(no contention)

# 802.11 – DCF with RTS/CTS (Review)

- Sending unicast packets
  - station can send RTS with reservation parameter after waiting for DIFS (reservation determines amount of time the data packet needs the medium)
  - acknowledgement via CTS after SIFS by receiver (if ready to receive)
  - sender can now send data at once, acknowledgement via ACK
  - other stations store medium reservations distributed via RTS and CTS



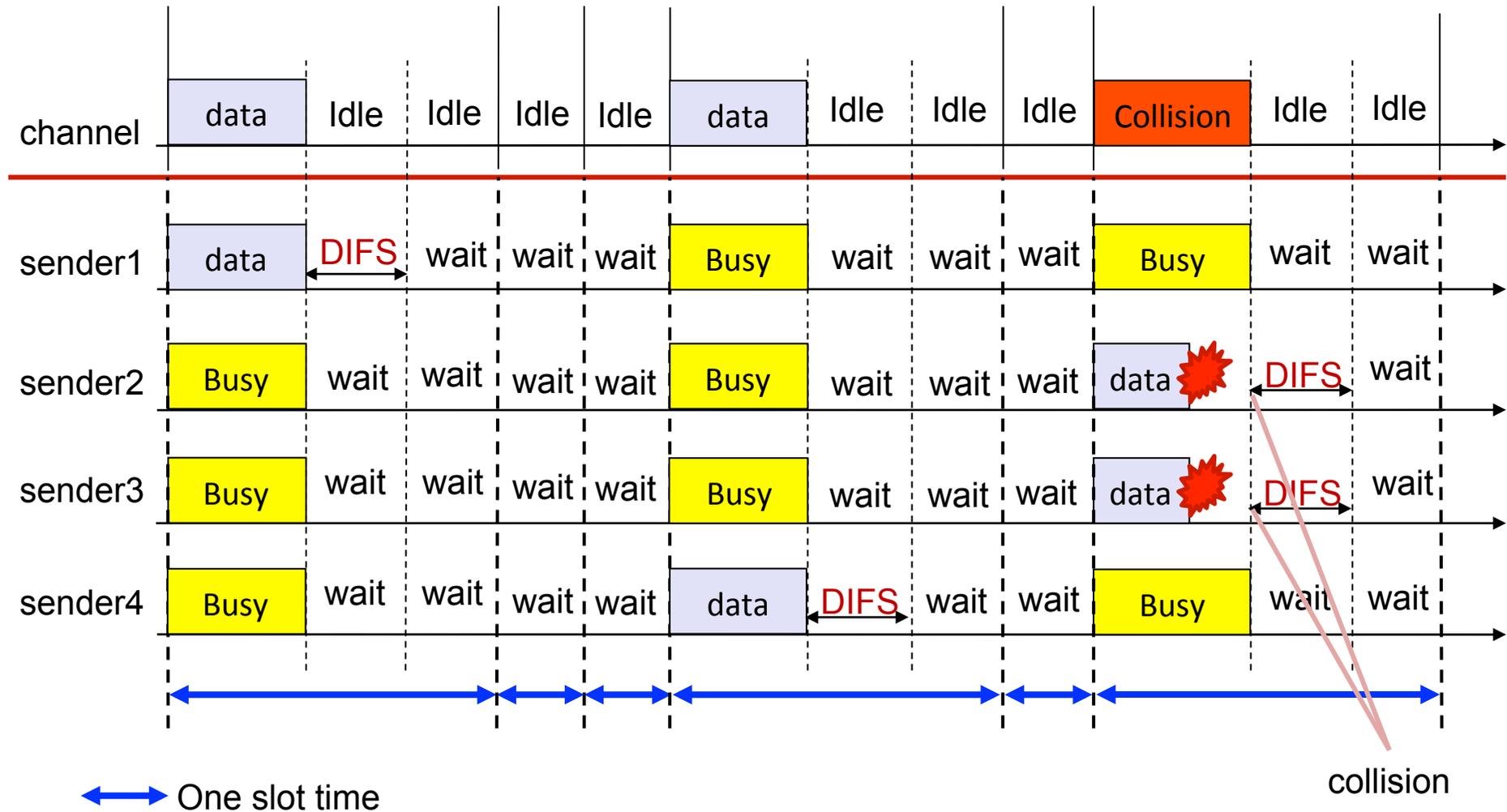
NAV: Net Allocation Vector

RTS/CTS can be present for some packets and not for other

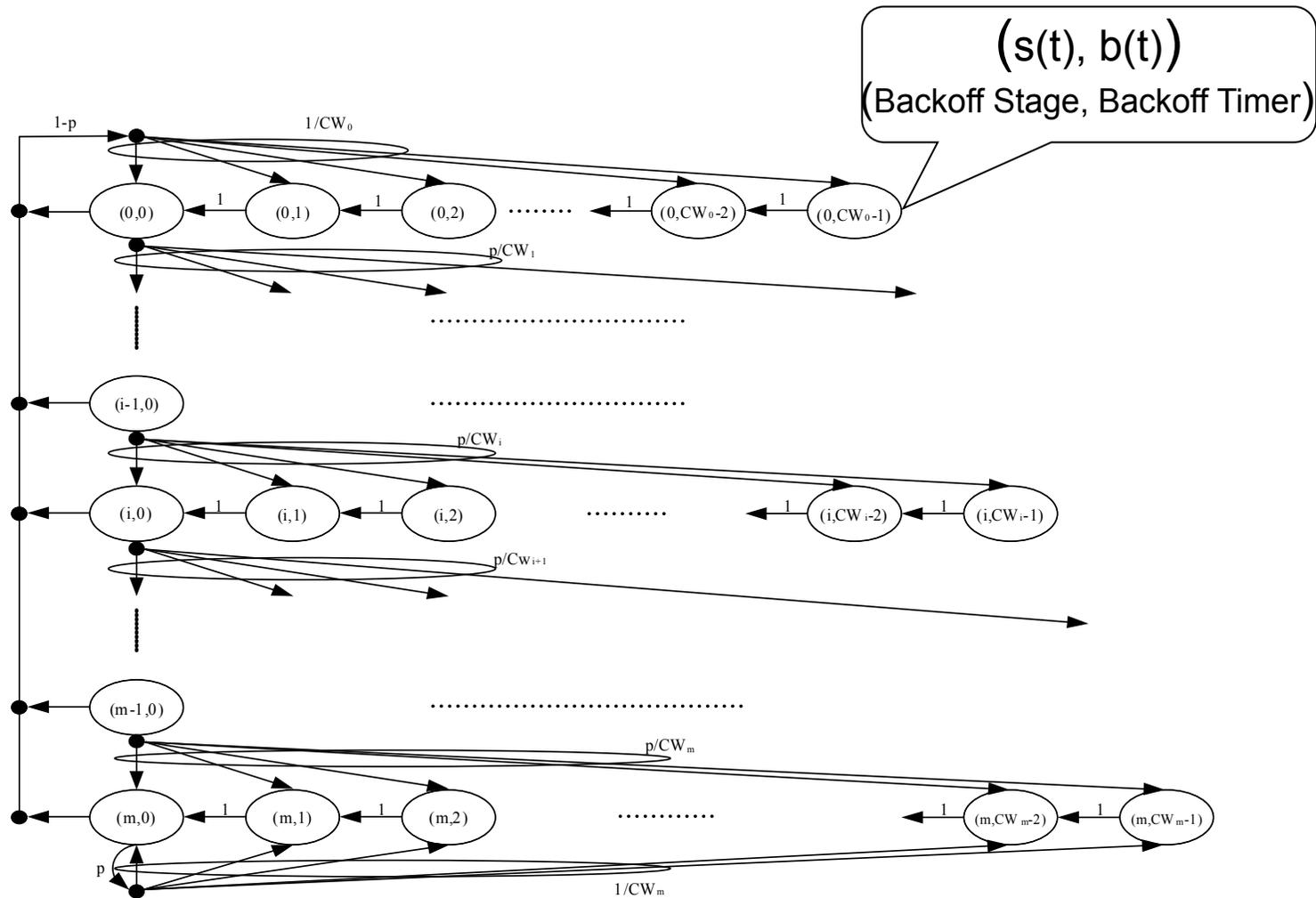
# Inter Frame Space and CW Times

<i>Parameters</i>	<i>802.11a</i>	<i>802.11b</i> <i>(FH)</i>	<i>802.11b</i> <i>(DS)</i>	<i>802.11b</i> <i>(IR)</i>	<i>802.11b</i> <i>(High Rate)</i>
<i>Slot Time (<math>\mu s</math>)</i>	9	50	20	8	20
<i>SIFS (<math>\mu s</math>)</i>	16	28	10	10	10
<i>DIFS (<math>\mu s</math>)</i>	34	128	50	26	50
<i>EIFS (<math>\mu s</math>)</i>	92.6	396	364	205 or 193	268 or 364
<i>CW<sub>min</sub>(SlotTime)</i>	15	15	31	63	31
<i>CW<sub>max</sub>(SlotTime)</i>	1023	1023	1023	1023	1023
<i>Physical Data Rate (Mbps)</i>	6 to 54	1 and 2	1 and 2	1 and 2	1, 2, 5.5, and 11

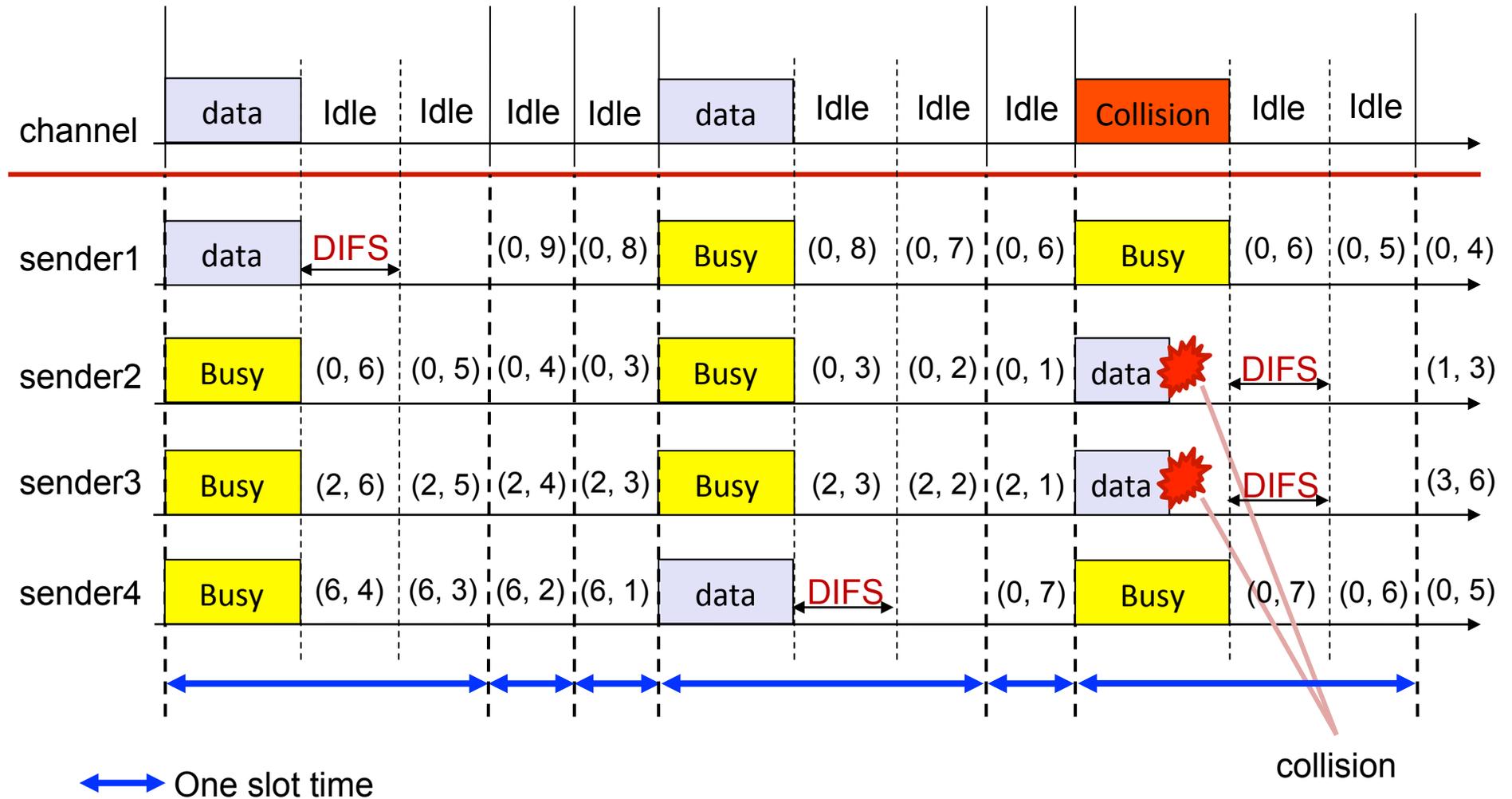
# 802.11 – Slot Time in Bianchi's Model



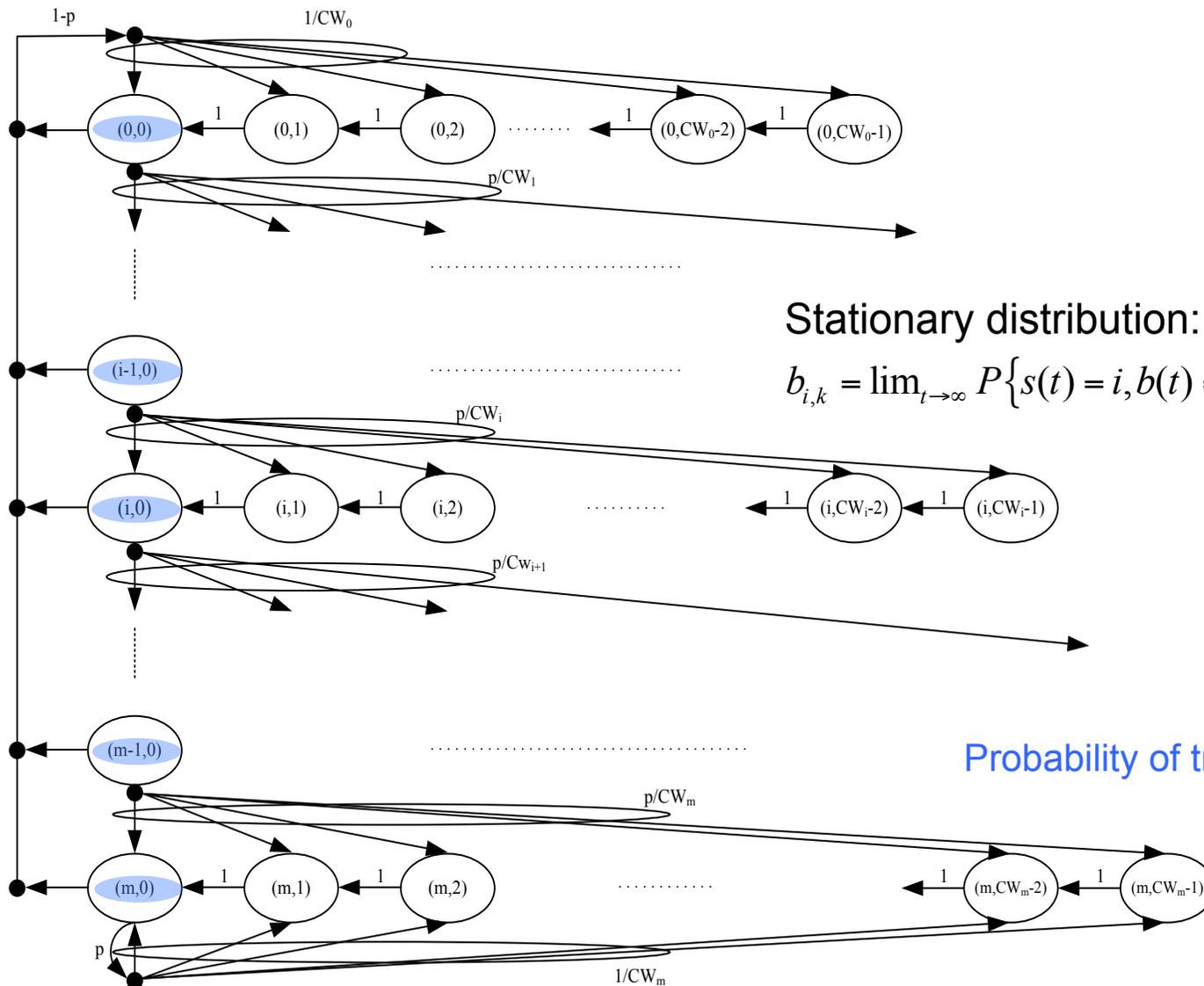
# Bianchi's Model: Two Dimensional Markov chain



# 802.11 – Slot Time in Bianchi's Model



# Bianchi's Model: Two Dimensional Markov chain



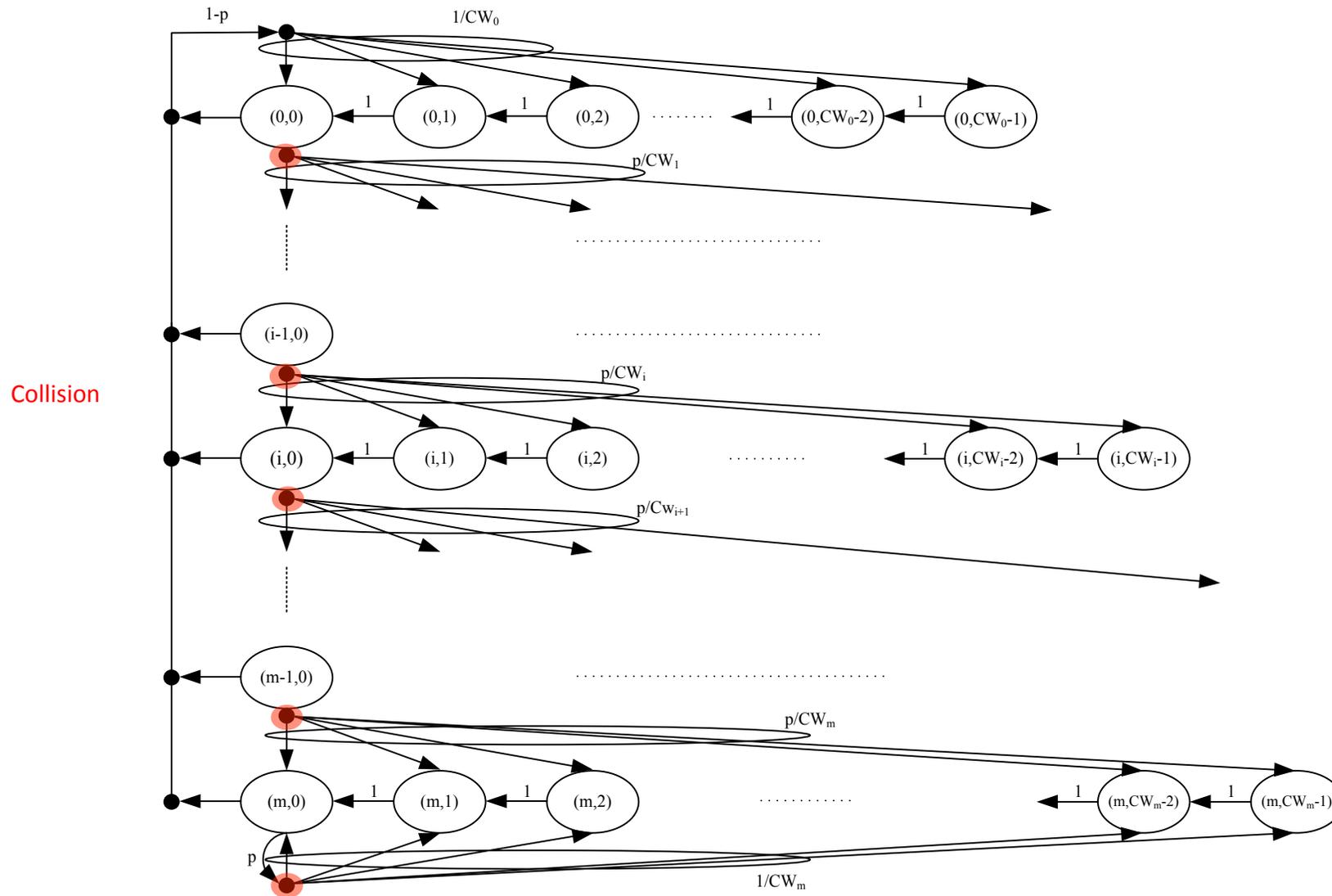
Stationary distribution:

$$b_{i,k} = \lim_{t \rightarrow \infty} P\{s(t) = i, b(t) = k\}, i \in (0, m), k \in (0, CW_i - 1)$$

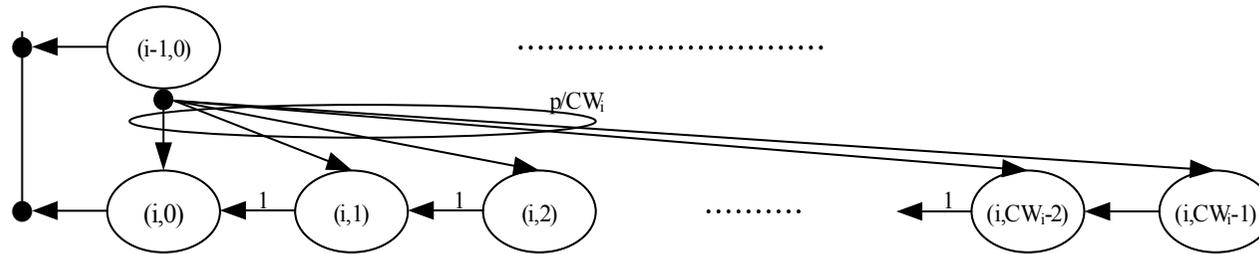
Probability of transmission:  $\pi = \sum_{i=0}^m b_{i,0}$



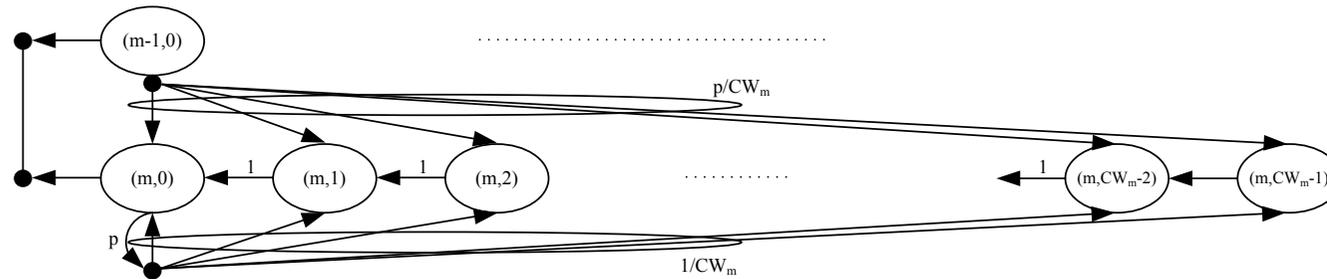
# Bianchi's Model: Two Dimensional Markov Chain



# Bianchi's Model: Stationary Distribution of Chain



$$b_{i,0} = p b_{i-1,0}$$



$$b_{m,0} = p b_{m-1,0} + p b_{m,0}$$

## Bianchi's Model: Solution for $p$ and $\pi$

After some derivations  $\rightarrow$  system of two nonlinear equations with two variables  $p$  and  $\pi$ :

$$\begin{cases} p = 1 - (1 - \pi)^{N-1} \\ \pi = \frac{2}{1 + W_{min} + pW_{min} \sum_{k=0}^{m-1} (2p)^k} \end{cases}$$

$\rightarrow$  Can be solved numerically to obtain  $p$  and  $\pi$

# Bianchi's model: Throughput Calculation

- Throughput of node i:

$$\tau_i = \frac{E[\text{Payload Transmitted by user } i \text{ in a slot time}]}{E[\text{Duration of slot time}]} = \frac{P_s P_{tr} L}{P_s P_{tr} T_s + P_{tr} (1 - P_s) T_c + (1 - P_{tr}) T_{id}}$$

- $P_{tr}$ : Probability of at least one transmission in slot time
- $P_s$ : Probability of successful transmission during a random time slot
- $L$ : Average packet payload size
- $T_s$ : Average time to transmit a packet of size  $L$
- $T_c$ : Average time of collision
- $T_{id}$ : Duration of the idle period
- $t_{ACK}$ : ACK transmission time
- $t_H$ : Header transmission time
- $t_L$ : Payload transmission time

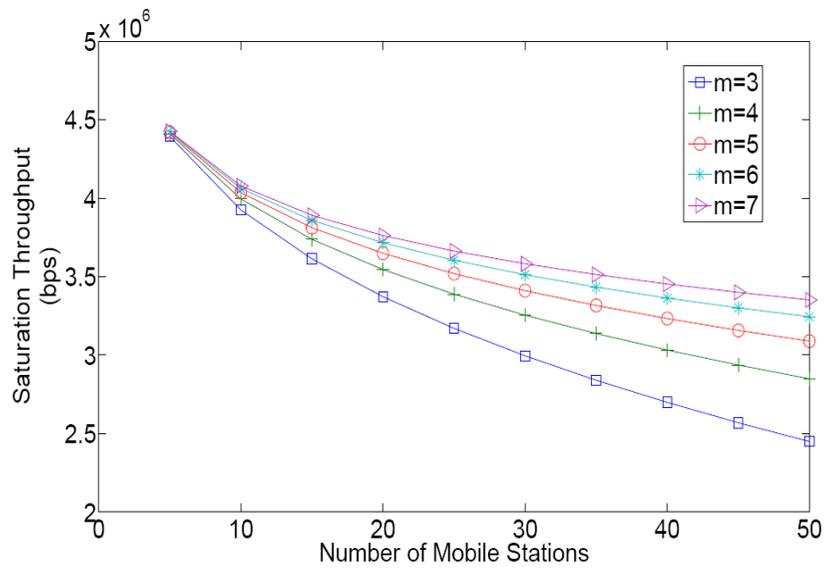
$$P_{tr} = 1 - (1 - \pi)^N$$

$$P_s = \frac{N\pi(1 - \pi)^{N-1}}{1 - (1 - \pi)^N}$$

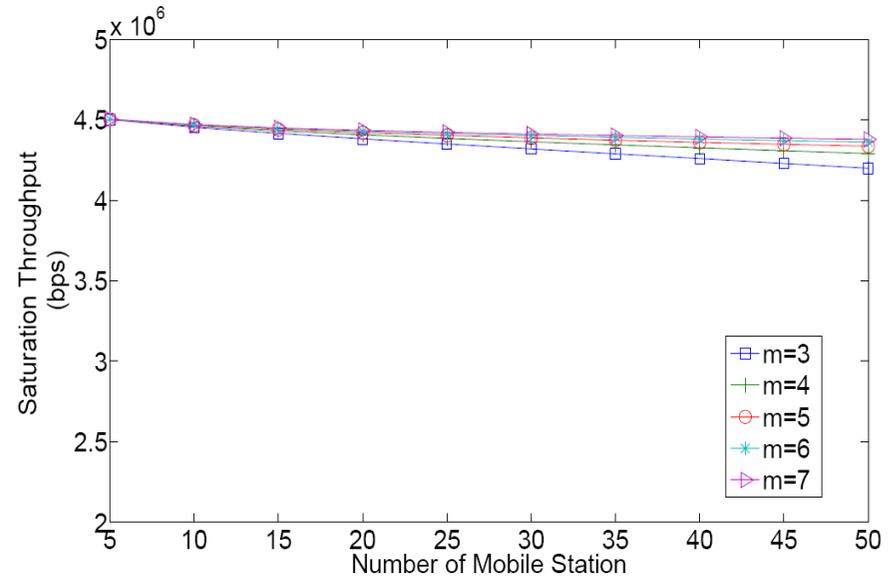
$$T_s = t_H + t_L + SIFS + \sigma + t_{ACK} + DIFS + \sigma$$

$$T_c = t_H + t_L + DIFS + \sigma$$

# Numerical Results



Basic Mode



RTS/CTS

# Conclusion

- Analytical model to express the performance of IEEE 802.11 networks
- More sophisticated models have been developed since then
- Don't forget checking the related write up: «Performance Analysis of the IEEE DCF: Bianchi Model»

# Performance Analysis of the IEEE 802.11 Distributed Coordination Function: Bianchi Model

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## 1 Introduction

Currently, IEEE 802.11 is the *de facto* standard for WLANs [1]. It specifies both the *medium access control* and the *physical* layers for WLANs. The scope of IEEE 802.11 *working groups* (WGs) is to propose and develop MAC and PHY layer specifications for WLAN to handle mobile and portable stations. In this standard, the MAC layer operates on top of one of several possible physical layers. Medium access is performed using *carrier sense multiple access with collision avoidance* (CSMA/CA). Concerning the physical layer, four IEEE 802.11 standards are available at the time of this writing: a, b, g, and n. The first IEEE 802.11 compliant products were based on 11b. Since the end of 2001, higher data rate products based on the IEEE 802.11a standard have appeared on the market [2]. The IEEE 802.11 working group has approved the 802.11g standard in June 2003, which extends the data rate of the IEEE 802.11b to 54 Mbps [3]. The 802.11g PHY layer employs all available modulations specified for 802.11a/b. IEEE 802.11n is a recent amendment which improves the data rate (up to to 600 Mbit/s) by adding multiple-input multiple-output (MIMO) and many other newer features.

In Section 2 of this document, we briefly describe the operating principles of the IEEE 802.11 MAC layer, which need to be known for a proper understanding of the IEEE 802.11 performance evaluation. There are different analytical models and simulation studies of the 802.11 MAC layer in saturated condition. In Section 3, we present one of the most well-known analytical model, the so-called Bianchi model [4], to analyze the performance of IEEE 802.11 MAC layer. Finally in Section 4, we present the numerical solution of Bianchi model for the 802.11a/b networks.

## 2 IEEE 802.11 MAC layer

The *distributed coordination function* (DCF) is the basic medium access mechanism of IEEE 802.11, and uses a *carrier sense multiple access with collision avoidance* (CSMA/CA) algorithm to mediate the access to the shared medium. The standard also describes centralized, polling-based access mechanism, the *point coordination function* (PCF) which is very rarely used in practice.

The DCF protocol in IEEE 802.11 standard defines how the medium is shared among stations. It includes a basic access method and an optional channel access method with *request-to-send* (RTS) and *clear-to-send* (CTS) exchanged as shown in Figure 1 and 2, respectively. First, we explain the basic access method.

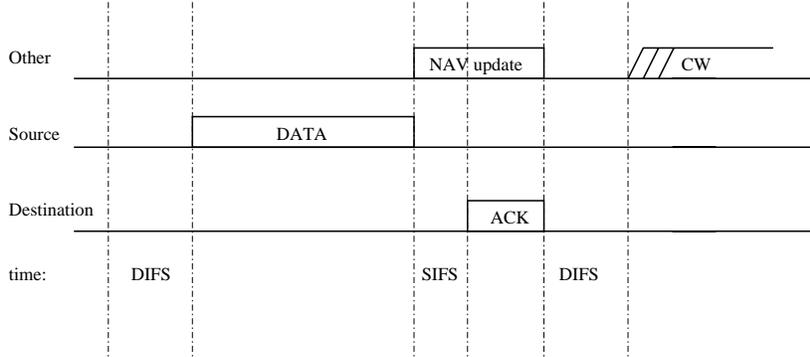


Figure 1: Basic access CSMA/CA protocol

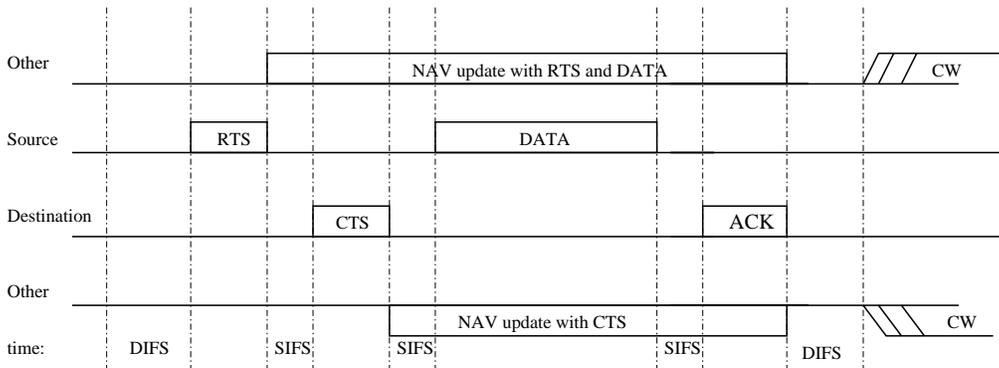


Figure 2: RTS/CTS exchange in the CSMA/CA protocol

If the channel is busy for the source, a backoff time (measured in slot times)<sup>1</sup> is chosen randomly in the interval  $[0, CW)$ , where  $CW$  stands for the *contention window*. This timer is decreased by one as long as the channel is sensed idle for a DIFS, i.e., *distributed inter-frame space* time. DIFS is equal to  $SIFS + 2 \times SlotTime$ , where SIFS stands for *short inter-frame space* (see values in Table 1). The timer stops when the channel is busy and resumes when the channel is idle again for at least a DIFS period.  $CW$  is an integer whose range is determined by the PHY layer characteristics:  $CW_{min}$  and  $CW_{max}$ .  $CW$  is doubled after each unsuccessful transmission, up to the maximum value equal to  $CW_{max} + 1$ .

When the backoff timer reaches zero, the source transmits the data packet. The ACK is transmitted by the receiver immediately after a period of duration equal to SIFS. When a data packet is transmitted, all other stations hearing this transmission adjust their *net allocation vector* (NAV). The NAV maintains a prediction of future traffic on the medium based on the duration information that is announced in Data frames (or RTS/CTS frames as will be explained in the following) prior

<sup>1</sup>The slot time is the sum of the Receiver-to-Transmitter turnaround time, MAC processing delay, and *clear channel assessment* (CCA) detect time [1]. The value of slot time for different PHY layer protocols is shown in Table 1.

to the actual exchange of data. In addition, whenever a node detects an erroneous frame, the node defers its transmission by a fixed duration indicated by EIFS, i.e., *extended inter-frame space* time. This time is equal to the  $SIFS + ACK_{time} + DIFS$  time.

The contention window is initially set to the minimum value of  $CW_{min}$ , equal for example to 15 (see Table 1). Every time a collision occurs, this is interpreted as a high load of the network, and each station involved in the collision throttles down its transmission rate by doubling the size of its contention window. In this way, the contention window can take values equal for example to 31, 63, 127, 255, 511, up to  $CW_{max} = 1023$ . Larger contention windows slow down the transmission of packets and reduce the probability of collisions. In case of a successful (i.e. collision-free) transmission, the transmitting station brings the value of its contention window back to  $CW_{min}$ . The mechanism we have just described is called *exponential backoff* or *binary exponential backoff*.

If the optional access method is used, an RTS frame should be transmitted by the source and the destination should accept the data transmission by sending a CTS frame prior to the transmission of the actual data packet. Note that stations in the sender’s range that hear the RTS packet should update their NAVs and defer their transmissions for the duration specified by the RTS. Nodes that overhear the CTS packet update their NAVs and refrain from transmitting. In this way, the transmission of the data packet and its corresponding ACK can proceed without interference from other nodes (hidden nodes problem). Table 1 shows the important time interval between frames in different standard specification called *inter-frame space* (IFS) [2, 5, 3]. IEEE 802.11g uses the IFS corresponding to its operating mode.

Table 1: Inter frame space and  $CW$  time for different PHY layers.

<i>Parameters</i>	<i>802.11a</i>	<i>802.11b</i> ( <i>FH</i> )	<i>802.11b</i> ( <i>DS</i> )	<i>802.11b</i> ( <i>IR</i> )	<i>802.11b</i> ( <i>High Rate</i> )
<i>Slot Time</i> ( $\mu s$ )	9	50	20	8	20
<i>SIFS</i> ( $\mu s$ )	16	28	10	10	10
<i>DIFS</i> ( $\mu s$ )	34	128	50	26	50
<i>EIFS</i> ( $\mu s$ )	92.6	396	364	205 or 193	268 or 364
$CW_{min}(SlotTime)$	15	15	31	63	31
$CW_{max}(SlotTime)$	1023	1023	1023	1023	1023
<i>Physical Data Rate</i> ( <i>Mbps</i> )	6 to 54	1 and 2	1 and 2	1 and 2	1, 2, 5.5, and 11

### 3 Bianchi Model

The main contribution of Bianchi’s model is the analytical calculation of saturation throughput in a closed-form expression. The model also calculates the probability of a packet transmission failure due to collision. It assumes that the channel is in ideal conditions, i.e., there is no hidden terminal and capture effect.

Bianchi uses a two-dimensional Markov chain of  $m + 1$  backoff stages in which each stage represents the backoff time counter of a node, see Figure 3. A transition takes place upon collision and successful transmission, to a “higher”<sup>2</sup> stage (e.g., from stage  $i - 1$  to stage  $i$  in Figure 3) and

<sup>2</sup>Actually it appears lower in the figure.

to the lowest stage (i.e., stage 0) respectively.

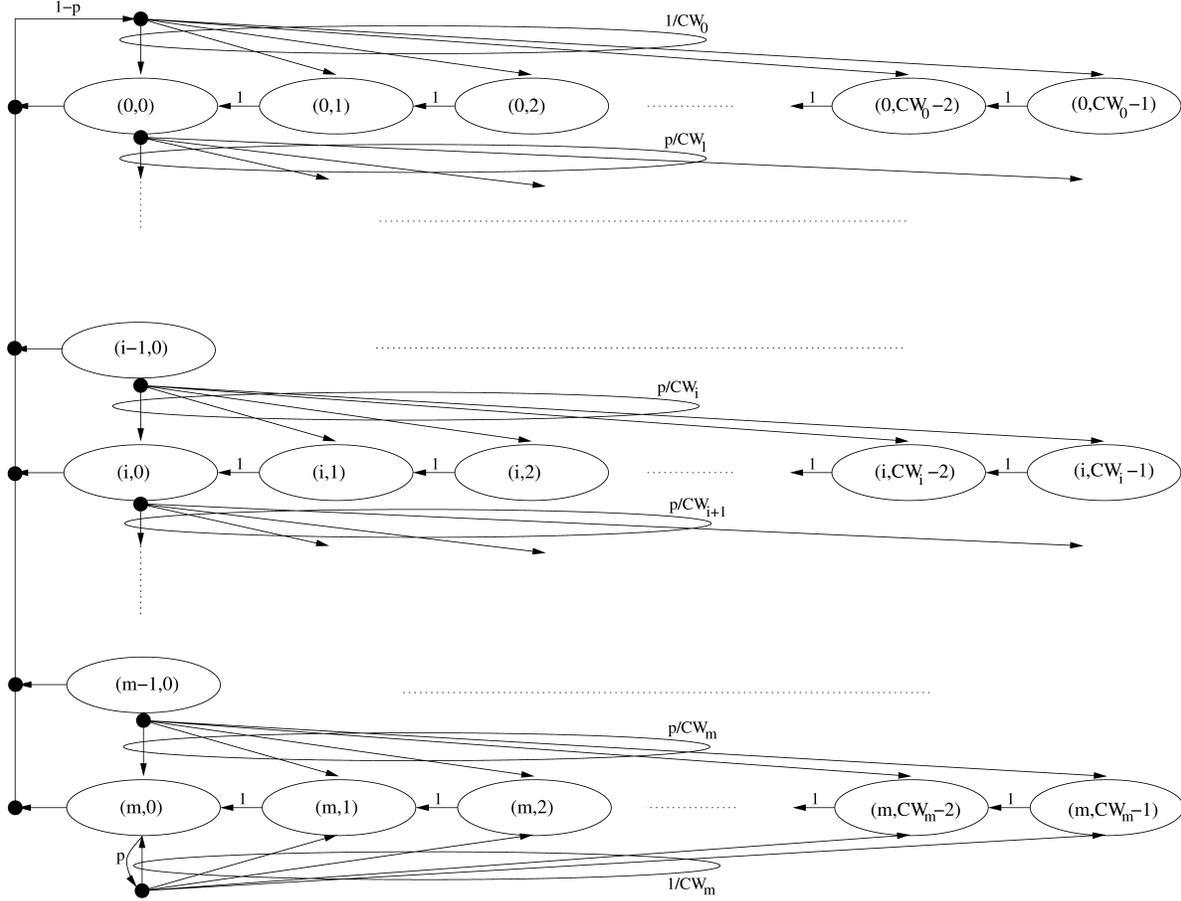


Figure 3: Markov chain model of backoff window size in CSMA/CA. In each stage,  $CW_i$  is the maximum value for the contention window and is equal to  $2^i(CW_{min} + 1)$  (Note that we define for convenience  $W_{min} = CW_{min} + 1$  and that  $CW_{max}$  is equal to  $2^m W_{min} - 1$ ). If a correct transmission takes place in any  $(i, 0)$  state, a random backoff will be chosen between 0 and  $CW_0 - 1$  with probability of  $\frac{1-p}{CW_0}$ . This case is represented by states  $(0, 0)$  to  $(0, CW_0 - 1)$  in the Markov chain. In the case of collision (e.g., in state  $(i - 1, 0)$ ), a random backoff will be chosen (between 0 and  $CW_i - 1$ , each with probability  $p/CW_i$ ). This case is represented by states  $(i, 0)$  to  $(i, CW_i - 1)$  in the Markov chain. From [4], © IEEE, 2000.

This model adopts a discrete and integer time scale. In this time scale,  $t$  and  $t + 1$  correspond to the beginning of two consecutive slot times. Each station decrements its backoff time counter at the beginning of each slot time. Note that as the backoff time decrement is stopped when the channel is busy, the time interval between  $t$  and  $t + 1$  may be much longer than the defined slot time for 802.11, as it may include a packet transmission or a collision.

Each state of this bidimensional Markov process is represented by  $\{s(t), b(t)\}$ , where  $b(t)$  is the stochastic process representing the backoff time counter for a given station and  $s(t)$  is the stochastic process representing the backoff stage  $(0, 1, \dots, m)$  of the station at time  $t$ . This model assumes

that in each transmission attempt, regardless of the number of retransmissions suffered, each packet collides with constant and independent probability  $p$ . In other words,  $p$  is the probability that, in a slot time, at least one of the  $N - 1$  remaining stations transmits as well. If at steady state each remaining station transmits a packet with probability  $\pi$ ,  $p$  can be written as:

$$p = 1 - (1 - \pi)^{N-1} \quad (1)$$

Let  $b_{i,k} = \lim_{t \rightarrow \infty} P\{s(t) = i, b(t) = k\}$ ,  $i \in (0, m)$ ,  $k \in (0, CW_i - 1)$  be the stationary distribution of the chain. A transmission occurs when the backoff time counter is equal to zero. Thus, we can write the probability that a station transmits in a randomly chosen slot time as:

$$\pi = \sum_{i=0}^m b_{i,0} \quad (2)$$

For the above Markov chain, it is easy to obtain a closed-form solution for  $b_{i,0}$  as a function of  $p$ . First, we can write the stationary distribution of the chain for  $b_{i,0}$ ,  $b_{m,0}$ , and  $b_{i,k}$ :

$$\begin{cases} b_{i,0} = p^i b_{0,0} & 0 < i < m \\ b_{m,0} = \frac{p^m}{1-p} b_{0,0} \\ b_{i,k} = \frac{CW_i - k}{CW_i} b_{i,0} & 0 \leq i \leq m, \quad 0 \leq k \leq CW_i - 1 \end{cases} \quad (3)$$

The first and second expressions in (3) account from the fact that  $b_{i-1,0} \cdot p = b_{i,0}$  for  $0 < i < m$  and  $b_{m-1,0} \cdot p = (1 - p)b_{m,0}$ . The third equation can be obtained considering the fact that  $\sum_{i=0}^m b_{i,0} = \frac{b_{0,0}}{1-p}$  and taking the chain regularities into account (for  $k \in (1, CW_i - 1)$ ), that is:

$$b_{i,k} = \frac{CW_i - k}{CW_i} \cdot \begin{cases} (1 - p) \sum_{j=0}^m b_{j,0} & i = 0 \\ p \cdot b_{i-1,0} & 0 < i < m \\ p \cdot (b_{m-1,0} + b_{m,0}) & i = m \end{cases} \quad (4)$$

By imposing the normalization condition and considering Equation (3), we can obtain  $b_{0,0}$  as a function of  $p$ :

$$\begin{aligned} 1 &= \sum_{i=0}^m \sum_{k=0}^{CW_i-1} b_{i,k} \\ &= \sum_{i=0}^m b_{i,0} \sum_{k=0}^{CW_i-1} \frac{CW_i - k}{CW_i} \\ &= \sum_{i=0}^m b_{i,0} \frac{CW_i + 1}{2} = \sum_{i=0}^m b_{i,0} \frac{2^i W_{min} + 1}{2} \\ &= b_{0,0} \frac{W_{min} + 1}{2} + \sum_{i=1}^{m-1} \left( b_{0,0} p^i \left( \frac{2^i W_{min} + 1}{2} \right) \right) + \left( \frac{b_{0,0} p^m}{1-p} \right) \left( \frac{2^m W_{min} + 1}{2} \right) \\ &= \frac{b_{0,0}}{2} \left[ W_{min} + 1 + \sum_{i=1}^{m-1} ((2p)^i W_{min} + p^i) + \frac{p^m}{1-p} (2^m W_{min} + 1) \right] \\ &= \frac{b_{0,0}}{2} \left[ W_{min} \left( \sum_{i=0}^{m-1} (2p)^i + \frac{(2p)^m}{1-p} \right) + \frac{1}{1-p} \right] \end{aligned} \quad (5)$$

Thus  $b_{0,0}$  can be written as:

$$b_{0,0} = \frac{2(1-2p)(1-p)}{(1-2p)(W_{min}+1) + pW_{min}(1-(2p)^m)} \quad (6)$$

Finally, considering equations (2), (3), and (6), the channel access probability  $\pi$  of a node is derived as a function of the number of backoff stage levels  $m$ , the minimum contention window value  $W_{min}$ , and the collision probability  $p$ :

$$\begin{aligned} \pi &= \sum_{i=0}^m b_{i,0} = \frac{b_{0,0}}{1-p} = \frac{2(1-2p)}{(1-2p)(W_{min}+1) + pW_{min}(1-(2p)^m)} \\ &= \frac{2}{1 + W_{min} + pW_{min} \sum_{k=0}^{m-1} (2p)^k} \end{aligned} \quad (7)$$

Equations (1) and (7) form a system of two nonlinear equations that has a unique solution and can be solved numerically for the values of  $p$  and  $\pi$  (e.g., one can use the *solve* function in MATLAB to obtain the values for  $p$  and  $\pi$ ). Once these probabilities are obtained, the saturation throughput, which is the average information payload transmitted in a slot time over the average duration of a slot time, can be computed as follows:

$$\begin{aligned} \tau &= \frac{E[\text{Payload information transmitted in a slot time}]}{E[\text{Duration of slot time}]} \\ &= \frac{P_s P_{tr} L}{P_s P_{tr} T_s + P_{tr}(1-P_s)T_c + (1-P_{tr})T_{id}} \end{aligned} \quad (8)$$

where  $P_{tr} = 1 - (1-\pi)^N$  is the probability that there is at least one transmission in the considered slot time;  $L$  is the average packet payload size;  $T_s$  is the average time needed to transmit a packet of size  $L$  (including the inter-frame spacing periods [4]);  $P_s = \frac{N\pi(1-\pi)^{N-1}}{1-(1-\pi)^N}$  is the probability of a successful transmission;  $T_{id}$  is the duration of the idle period (a single slot time); and  $T_c$  is the average time spent in the collision.

$T_c$  and  $T_s$  can be calculated for the basic transmission mode (i.e., no RTS and CTS packets) with:

$$\begin{cases} T_s = H + L + SIFS + \sigma + ACK + DIFS + \sigma \\ T_c = H + L + DIFS + \sigma \end{cases} \quad (9)$$

where  $H$ ,  $L$ , and  $ACK$  are the transmission times needed to send the packet header, the payload, and the acknowledgment, respectively.  $\sigma$  is the propagation delay. Note that for RTS/CTS transmission mode,  $T_c$  and  $T_s$  can be calculated by:

$$\begin{cases} T_s = RTS + SIFS + \sigma + CTS + SIFS + \sigma + H + L + SIFS + \sigma + ACK + DIFS + \sigma \\ T_c = RTS + DIFS + \sigma \end{cases} \quad (10)$$

## 4 Numerical Solution

In this section we present the numerical solution results of the Bianchi model. We use MATLAB to solve the two nonlinear equations (1) and (7) numerically. The system values are those specified for the DSSS 802.11b and 802.11a in Table 1. The channel bit rate has been assumed equal to 1 and 6 *Mbps* for 802.11b and 802.11a respectively. We assume that the packet payloads to be transmitted are all 1000-octet long. Figure 4 and 5 show the saturation throughput of IEEE 802.11a and 802.11b networks with basic transmission mode and with RTS/CTS packets, using the Bianchi model. Each curve correspond to a different value of the maximum backoff stage, i.e.,  $m$ .

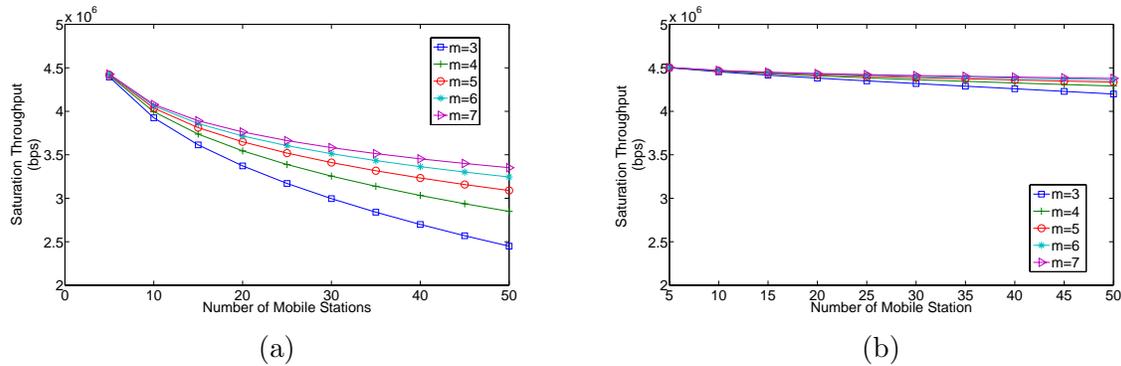


Figure 4: Saturation throughput for 802.11a for 6 Mbps physical data rate: (a) Basic transmission mode, (b) RTS/CTS transmission mode.

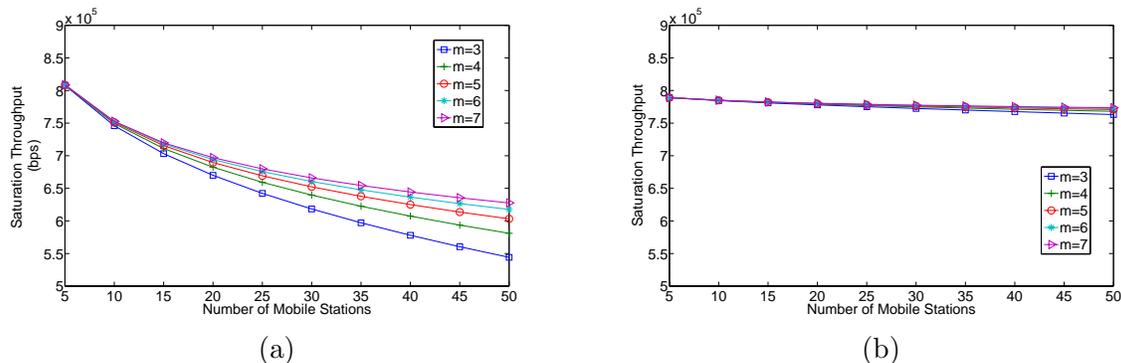


Figure 5: Saturation throughput for 802.11b DSSS for 1 Mbps physical data rate: (a) Basic transmission mode, (b) RTS/CTS transmission mode.

As expected, the RTS/CTS transmission modes show better throughput performance for the high number of mobile stations as it avoids the collision between the long data packets. Another interesting observation is that the saturation throughput increases for the higher maximum backoff stages. Note that the Bianchi model does not take into consideration the retransmission limit and the maximum backoff stage as defined by the IEEE standard specification [1].

In 2002, Wu et al. [6] proposed a refinement of Bianchi's model by considering finite packet

retry limits. The retransmission limit is defined in the IEEE 802.11 MAC standard specification with the help of the two following counters: *short retry count* (SRC) and *long retry count* (LRC). These counters are incremented and reset independently. SRC is incremented every time an RTS fails whereas LRC is incremented when data transmission fails. Both SRC and LRC are reset to zero after a successful data transmission. Data frames are discarded when LRC (SRC) reaches *dot11LongRetryLimit* (*dot11ShortRetryLimit*). The default values for *dot11LongRetryLimit* and *dot11ShortRetryLimit* are 4 and 7 respectively. Considering this limitation, the Markov chain proposed by Bianchi is modified in [6]. Unlike Bianchi model [4], in the Wu's model,  $m$  is the maximum backoff stage (retransmission count) which is different for the data and RTS frames. In Wu's model,  $m'$  represents the maximum contention window, i.e.  $2^{m'}(CW_{min} + 1) = (CW_{max} + 1)$ . In fact, the key difference between Bianchi's model [4] and Wu's model [6], concerns the Markov chain models. They are different because Wu's model considers the effects of frame retransmission limit. The interested reader is referred to [6] for more information.

## References

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