بسم الله الرحمن الرحيم

Foundations of Game Theory for Electrical and Computer Engineering

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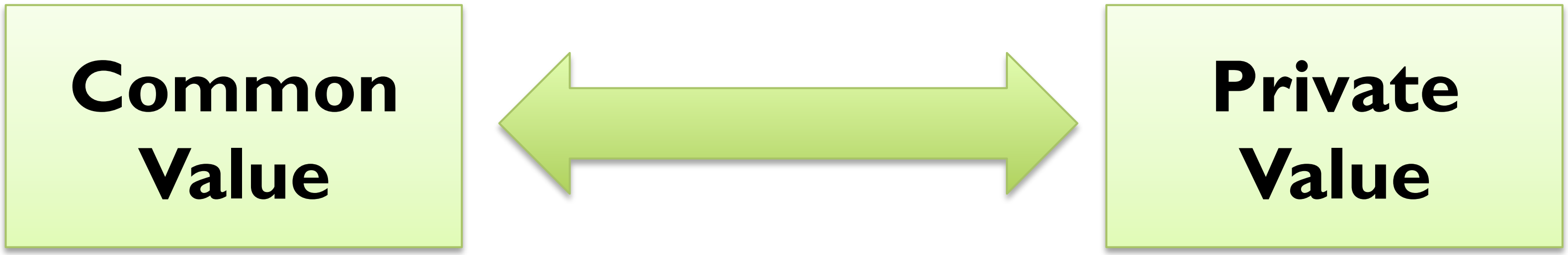
Change the World!

AUCTION THEORY

Auction

You compete with others but
you do not know the value of good for others



Value of good for sale is same of all [V]

Value of good is different for all and my value is irrelevant to you [V_i]

Auction

Let's make an Auction!

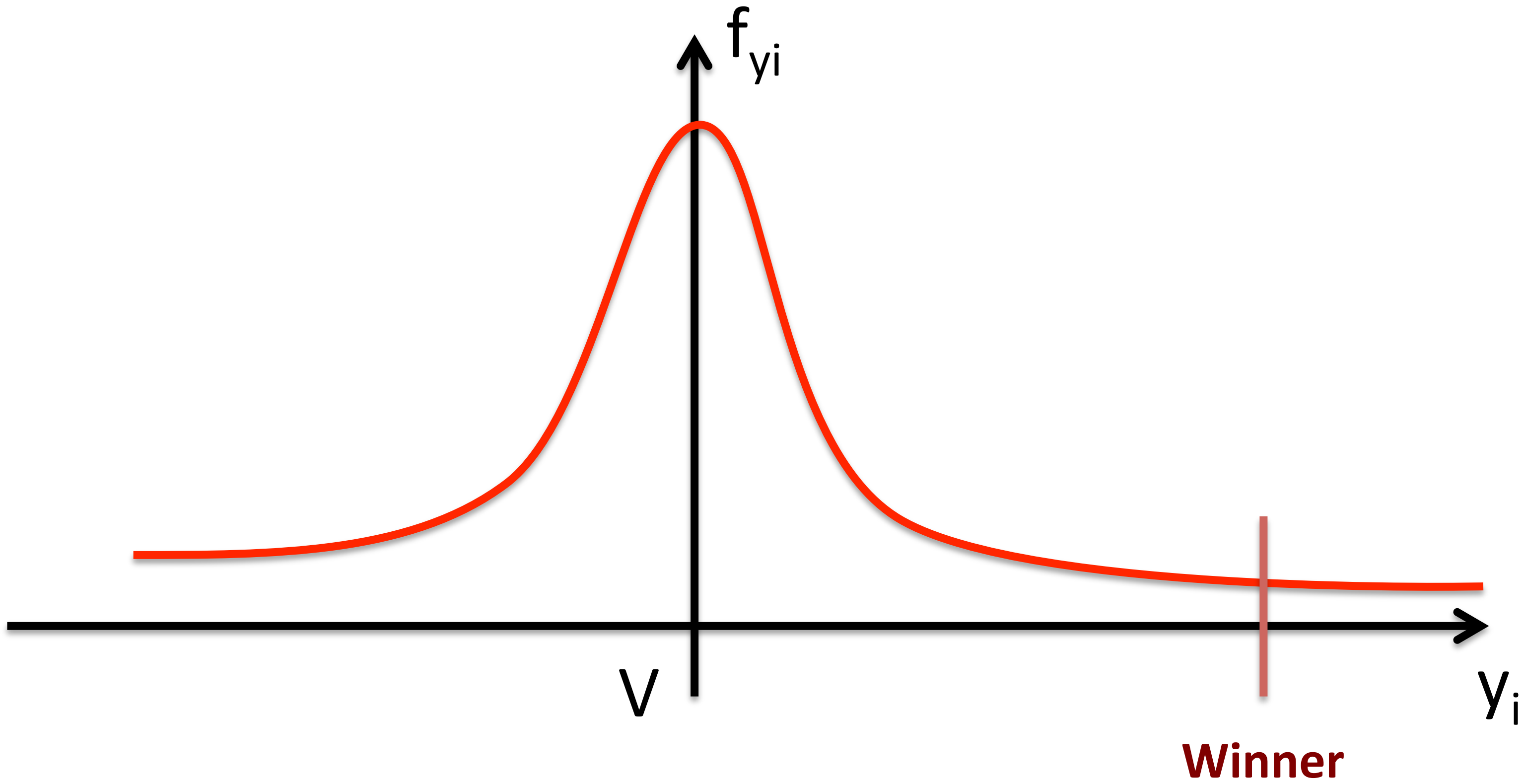
Winning Bid >> True Value
Winner's Curse

Payoff in Auction

$$\text{Payoff in the auction} = \begin{cases} V - b_i & \text{If you are the highest} \\ 0 & \text{Otherwise} \end{cases}$$

Your Estimate: $y_i = V + \varepsilon_i$

Who is the Winner?



Your Estimate: $y_i = V + \varepsilon_i$

Winner is the i whose y_i is Max, i.e., ε_i is Max

(On Average) the winning bid >> V

How to bid?

- When you win, you will learn that
  - $y_j < y_i$ for all j
- The relevant estimate when biding is:
  - What is the value of the good, given y_i (My Guess), and given $y_i > y_j$ for all j.

- Main Lesson:
  - Bid as if you know you win, then you won't regret winning

Types of Auctions

A. First-price Sealed Bid
B. Second Price Sealed Bid
   - ✧ Winner pays the second bid (Vickery)
C. Ascending Open Auction (eBay)
D. Descending Open Auction (Dutch)

D ≈ A   B ≈ C

Private Value Auction

- In **second-price sealed bid or open ascending**
  - Value $\mathbf{v_i}$ and bid $\mathbf{b_i}$, the payoff is,

$$\begin{cases} \mathbf{v_i - \underline{b}_j} & \text{if } \mathbf{b_i} \text{ is the highest and } \mathbf{\underline{b}_j} \text{ is the highest other bid} \\ \mathbf{0} & \text{o.w.} \end{cases}$$

  - Setting $\mathbf{b_i = v_i}$ is weakly dominant

Proof of Dominance in Truthful Bidding

Payoff for bidder $= \begin{cases} v_i - \underline{b}_j & \text{if } b_i \text{ is the highest and } \underline{b}_j \text{ is the highest other bid} \\ 0 & \text{o.w.} \end{cases}$

Proof (Part 1):

let's assume that $b_i > v_i$

1. **If $\underline{b}_j < v_i$:** The bidder would win the item with a truthful bid as well as an overbid. The bid's amount does not change the payoff so the two strategies have equal payoffs.

2. **If $\underline{b}_j > b_i$:** The bidder would lose the item either way so the strategies have equal payoffs.

3. **If $v_i < \underline{b}_j < b_i$: T**he strategy of overbidding would win the auction. The payoff would be negative for the strategy of overbidding because they paid more than their value of the item, while the payoff for a truthful bid would be zero. Thus the strategy of bidding higher than one's true valuation is dominated by the strategy of truthfully bidding.

Proof of Dominance in Truthful Bidding

Payoff for bidder $\begin{cases} v_i - \underline{b}_j & \text{if } b_i \text{ is the highest and } \underline{b}_j \text{ is the highest other bid} \\ 0 & \text{o.w.} \end{cases}$

Proof (Part 2):

let's assume that $b_i < v_i$

1. **If** $v_i < \underline{b}_j$ **:** The bidder would lose the item with a truthful bid as well as an underbid, so the strategies have equal payoffs for this case.

2. $\underline{b}_j < b_i$**:** The bidder would win the item either way so the strategies have equal payoffs.

3. **If** $b_i < \underline{b}_j < v_i$**:** The strategy of truthfully bidding would win the auction. The payoff for the truthful strategy would be positive as they paid less than their value of the item, while the payoff for an underbid bid would be zero. Thus the strategy of underbidding is dominated by the strategy of truthfully bidding.

Private Value Auction

- In **the first-price auction** the payoff is

$$\begin{cases} \mathbf{v_i - b_i} & \text{if win} \\ \mathbf{0} & \text{o.w.} \end{cases}$$

  - Bidding your value is weakly dominated