Foundations of Game Theory for Electrical and Computer Engineering

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Let's use Game Theory!

MECHANISM DESIGN: STABLE MATCHING

Example

Adam
basketball > Walk > Karting

Karting

Bob
basketball > Walk > Karting

Basketball

Carl
Walk > Karting > basketball

David
Karting > Walk > basketball

Walk

We announce that we will select the activity with
the highest number of votes, breaking ties **reverse alphabetically**

But David Can Cheat!!

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- Is there anything you can do to prevent such manipulation by agents?

- This is where *mechanism design*, or *implementation theory* comes in!

- It is sometimes called "inverse game theory."

# Social Choice Theory vs Mechanism Design

- *Social choice theory* is nonstrategic; it takes the preferences of the agents as given, and investigates ways in which they can be aggregated.

- *Mechanism design* is a strategic version of social choice theory, which adds the assumption that agents will behave so as to maximize their individual payoffs.

Game Theory:
Analysis vs. Mechanism Design

analysis

system   behavior

mechanism design

Algorithmic Mechanism Design

system
(protocols)

behavior
(truthful)
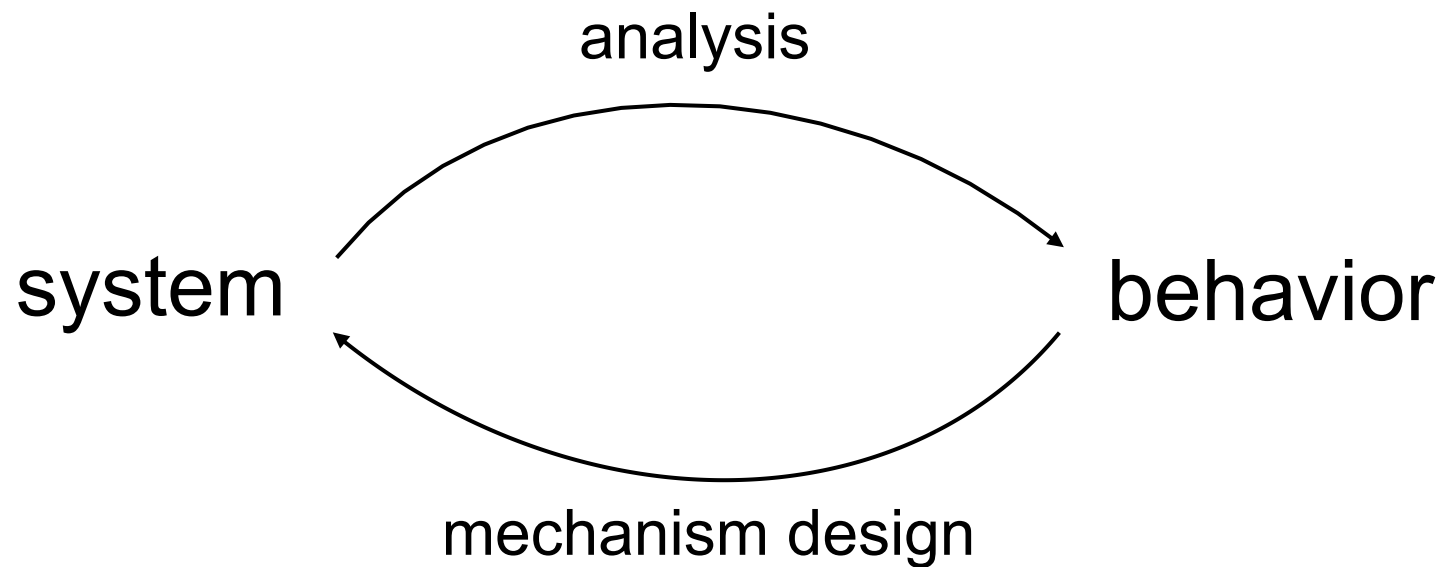
algorithmic
mechanism design
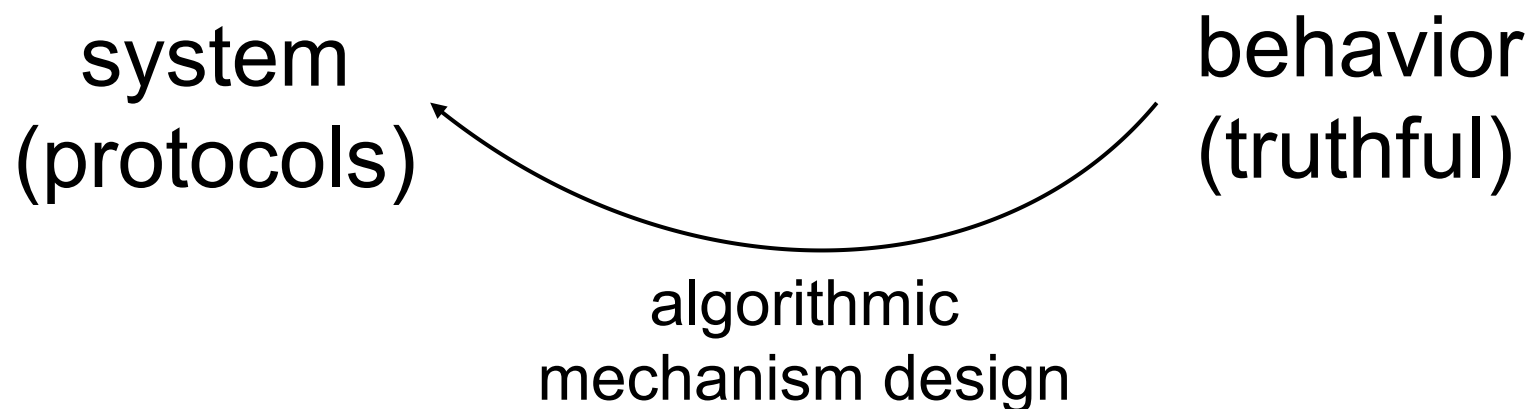
Marriage Stable Matching

Adam — Geeta, Heiki, Irina, Fran

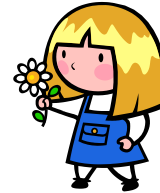Bob — Irina, Fran, Heiki, Geeta

Carl — Geeta, Fran, Heiki, Irina

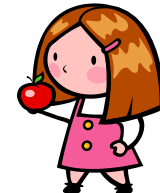David — Irina, Heiki, Geeta, Fran

Fran — Adam, Bob, Carl, David

Geeta — Carl, David, Bob, Adam

Heiki — Carl, Bob, David, Adam

Irina — Adam, Carl, David, Bob

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Search for a Matching

Adam   Geeta

David   Heiki

Geeta prefers Carl to Adam!

Blocking Pair

Bob   Irina

Carl   Fran

Carl likes Geeta better than Fran!
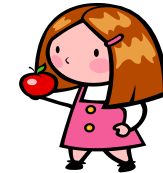
Stable Matching

Adam ~~~ Heiki

David ~~~ Irina

Unfortunately,
Irina loves David better!

Stable Matching: a matching without blocking pairs

Bob and Irina are not a blocking pair

Bob ~~~ Fran

Carl ~~~ Geeta

Bob likes Irina better than Fran!
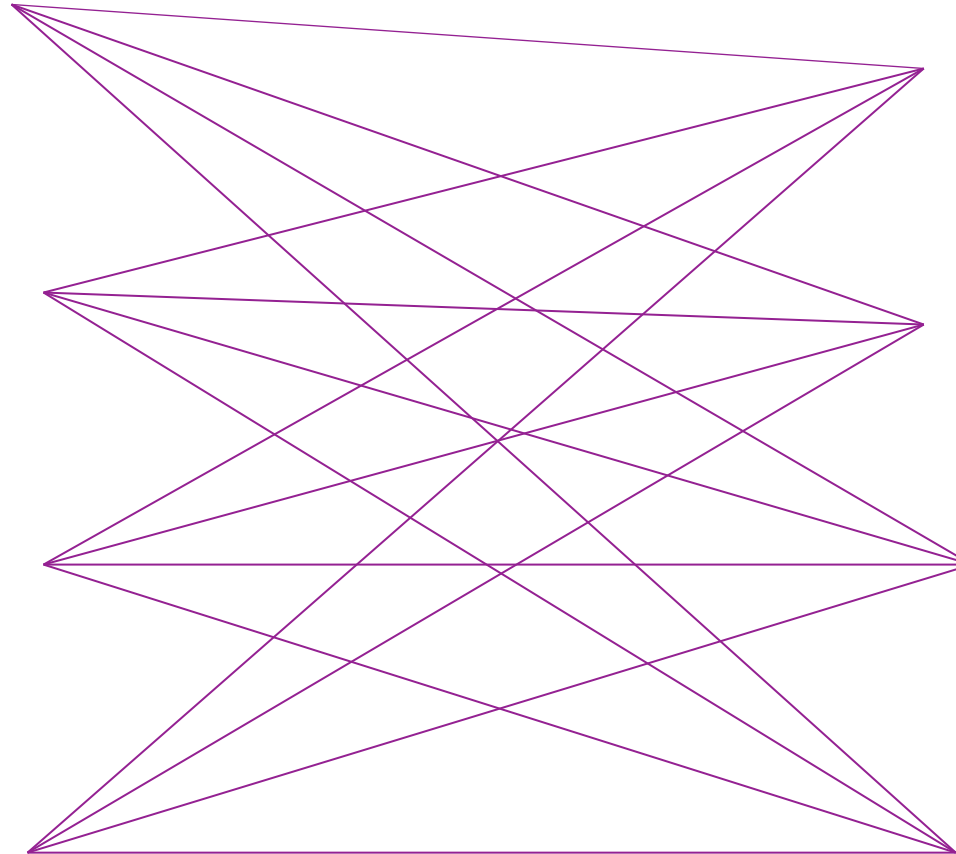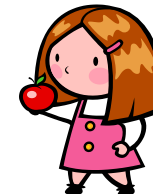
Goal

Adam Bob Carl David

Fran Geeta Heiki Irina

A matching is stable if and only if it is individually rational and unblocked.

A Stable Matching: Application

- **(Gale and Shapley, 1962)** *A stable matching always exists.*

- Gale-Shapley Stable Matching algorithm
  - **Men Propose, women accept/reject**

- The first algorithm for finding stable matchings was developed by Stalnaker [1953], and was used to match medical interns to hospitals.

[1] Stalnaker, J.M. (1953). The matching program for intern replacement: The second year of operation. *Journal of Medical Education*, *28*, 13–19.
[2] Roth, A. E. (1984). The evolution of the labor market for medical interns and residents: A case study in game theory. *Journal of Political Economy*, *92*, 991–1016.

Gale-Shapley Algorithm

Adam — Geeta, Heiki, Irina, Fran

Bob — Irina, Fran, Heiki, Geeta

This is a stable matching

Carl — Geeta, Fran, Heiki, Irina
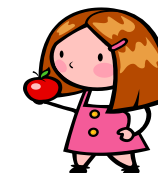
David — Irina, Heiki, Geeta, Fran

Fran

Geeta — Carl > Adam

Heiki

Irina — David > Bob

Cheating in the Gale-Shapley Stable Matching

- Can men cheat?

  - For men, individually, being truthful is a dominant strategy

- But women can cheat!

Acceptance Algorithm: Student-Application Version

- **Step 1:** *each student applies to his most preferred advisor.*

- **repeat**
  - **Step 2:** *each advisor keeps her most preferred acceptable application (if any) and rejects the rest (if any).*

  - **Step 3:** *each student who was rejected at the previous step applies to his next acceptable choice.*

- **until** *no student applied in the last step*
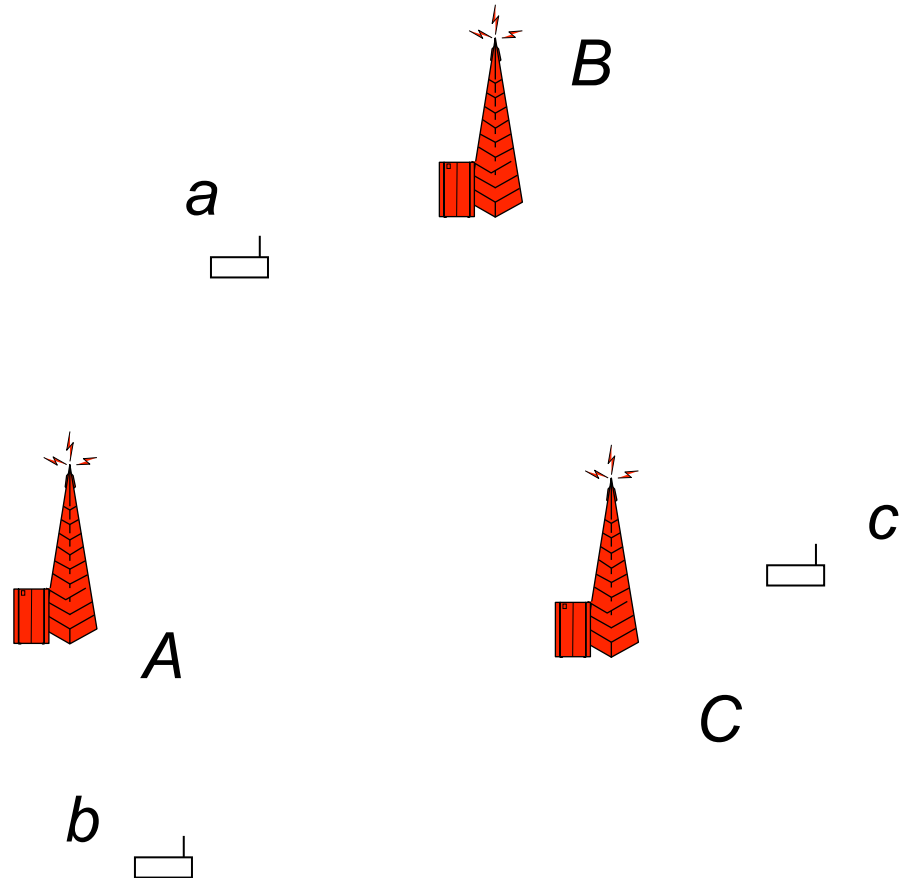
Stable Matching
in Cellular Networks (CSM)
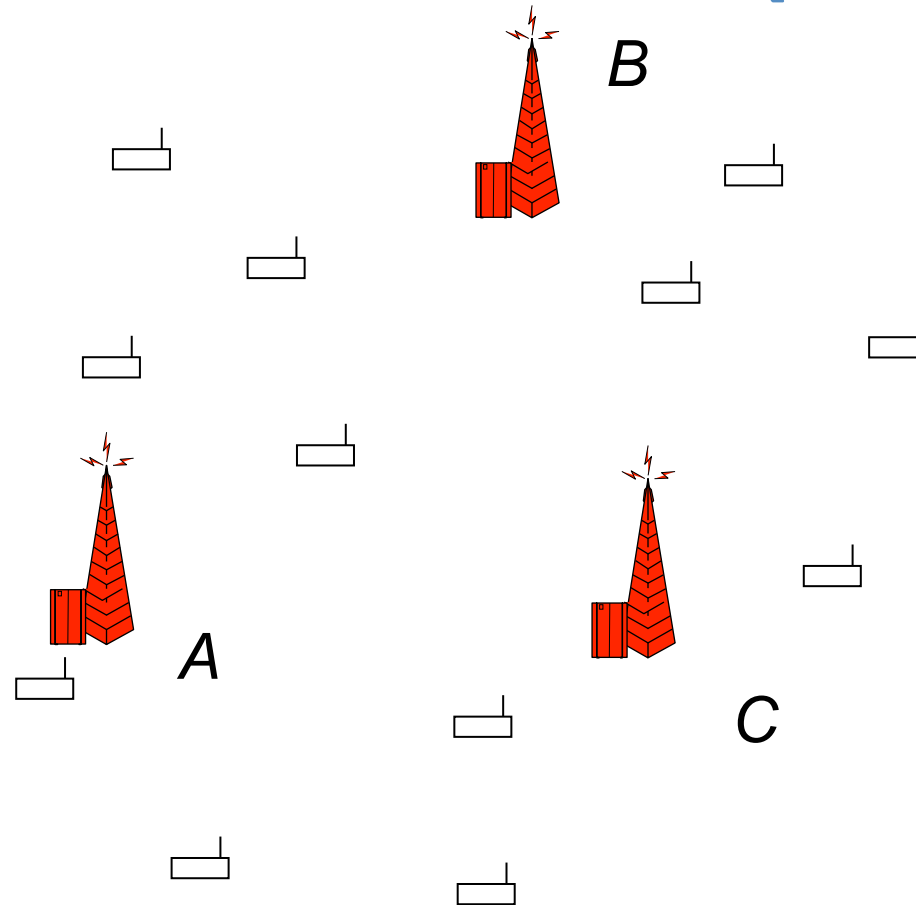
Cellular Stable Matching (CSM)

- N base stations and M mobiles (M>N)
- Each BS and mobile has preference lists
- **Unstable**: If BS *A* and *B* serve mobile *a* and *b*, respectively, although *a* prefers *B* and *B* also prefers *a.*
- Simplified version: N=M

$$\downarrow$$

- Stable marriage problem

Cellular Stable Matching Simplified (CSM')

- N=M

B

a

A

c

C

b

Cellular Stable Matching Simplified

- Preference Matrix

BS \ mobiles	a	b	c
A	1,3	2,2	3,1
B	3,1	1,3	2,2
C	2,2	3,1	1,3

Existence of CSM

- **Theorem (Gale-Shapley, 1962):** There always exists a stable matching

- Note that the proof is constructive: The Gale-Shapley algorithm

CSM – The Gale-Shapley Algorithm

BS \ mobiles	a	b	c
A	1,3	2,2	3,1
B	3,1	1,3	2,2
C	2,2	3,1	1,3

network-centric

user-centric

1 - A ------------- a - 3
1 - B ------------- b - 3
1 - C ------------- c - 3

3 - A ------------- a - 1
3 - B ------------- b - 1
3 - C ------------- c - 1
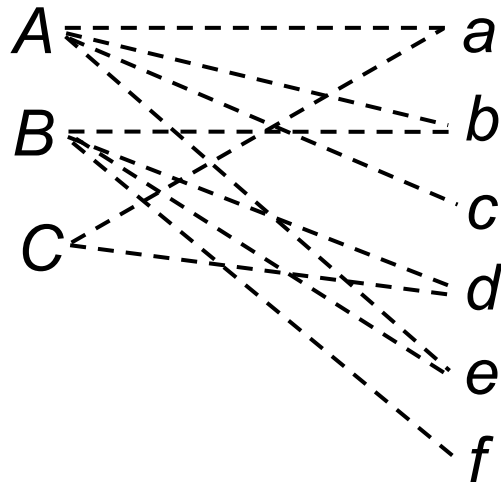
Optimality of CSM

- **Optimal:** Each participant (in a group) is at least as well off as in another matching

- The Gale-Shapley algorithm is male (BS)-optimal

Back to CSM

- The Gale-Shapley algorithm generalized
  - quota at each BS (q=2)

mobiles

BS	a	b	c	d	e	f
A	1,3	2,2	4,3	6,1	3,2	5,1
B	6,1	1,3	5,1	3,3	4,1	2,2
C	2,2	3,1	6,2	1,2	5,3	4,3

A   a

B   b

         c

C   d

   e

         f

Stable Matching

- **Applications (Eng.):**
  - WiFi networks
  - Load-balancing for processors
  - Bandwidth Allocation
  - Task Scheduling
  - Switching

- **Applications (Others):**
  - Students to schools
  - Job-hunting
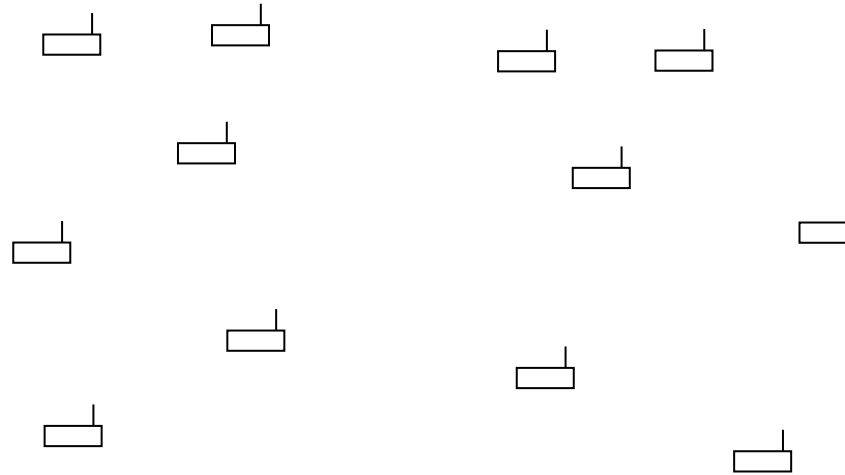
Truthful Stable Matching

- **Theorem (Roth, 1982):**
  - The Gale-Shapley algorithm is truthful for males
- **Theorem (Gale-Sotomayor, 1985):**
  - Women can cheat such that they get a better partner

mobiles

BS	a	b	c
A	1,3	2,2	3,1
B	3,1	1,3	2,2
C	2,2	3,1	1,3
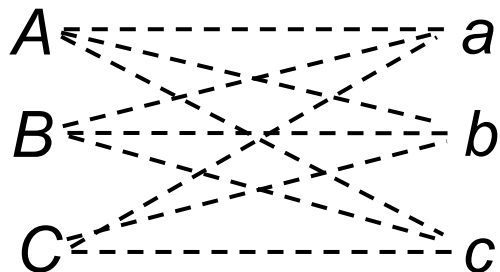
- Accept only your favorite peer
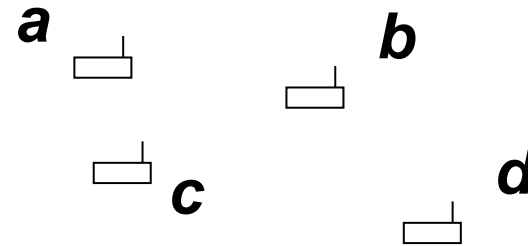
Stable Matching in Ad Hoc Networks (ASM)

- M mobiles
- Each mobile has preference lists
- **Unstable**: If mobile *a* and *c* communicate with mobile *b* and *d*, respectively, although *a* prefers *c* and *c* prefers *a.*

  ↓

- Stable roommate problem

ASM Simplified

- **preference matrix**

a b c d

mobiles

mobiles

	a	b	c	d
a	-	1,3	2,2	3,1
b	3,1	-	1,3	2,2
c	2,2	3,1	-	1,3
d	1,3	2,2	3,1	-

a ——— b

c ——— d