بسم الله الرحمن الرحیم

Foundations of Game Theory for Electrical and Computer Engineering

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EVOLUTIONARY STABILITY

Introduction to Evolution

- Concept related to a specific branch of Biology
- Relates to the evolution of the species in nature
- Powerful modeling tool that has received a lot of attention lately by **the computer science community**

- Why look at evolution in the context of Game Theory?

Game Theory Helps Biology

- Game Theory had a tremendous influence on evolutionary Biology
- Study animal behavior and use GT to understand population dynamics
- Idea:
  - Relate strategies to phenotypes of genes
  - Relate payoffs to genetic fitness
  - Strategies that do well grow, those that obtain lower payoffs die out
- Important note:
  - Strategies are ***hardwired***

Examples (Bio and Eng)

- Examples:
  - Group of lions deciding whether to attack in group an antelope
  - Ants deciding to respond to an attack of a spider

  - **Mobile ad hoc networks**
  - **TCP variations**
  - **P2P applications**

Biology Helps Game Theory

- Evolutionary biology had a great influence on Game Theory
- Similar ideas as before, relate strategies and payoffs to genes and fitness
- Example:
  - Firms in a competitive market
  - Firms are bounded, they can't compute the best response, but have rules of thumbs and adopt hardwired (consistent) strategies

  ➔ Survival of the fittest == rise of firms with low costs and high profits

Simplifying Assumptions

- When studying evolution through the lenses of GT, we need to make some assumptions to make our life easy
  - We can relax these assumptions later on

1. Within species competition
   - We assume no mixture of population: ants with ants, lions with lions
2. Asexual reproduction
   - We assume no gene redistribution

Evolutionary Game Theory
A Simple Model

- We will look at simple games at first
  - Two player symmetric games: all players have the same strategies and the same payoff structure
- We will assume random tournaments
  - In a large population of individuals, we pick two individuals at random and we make them play the symmetric game
  - The player adopting the strategy yielding higher payoff will survive (and eventually gain new elements) whereas the player who "lost" the game will die out

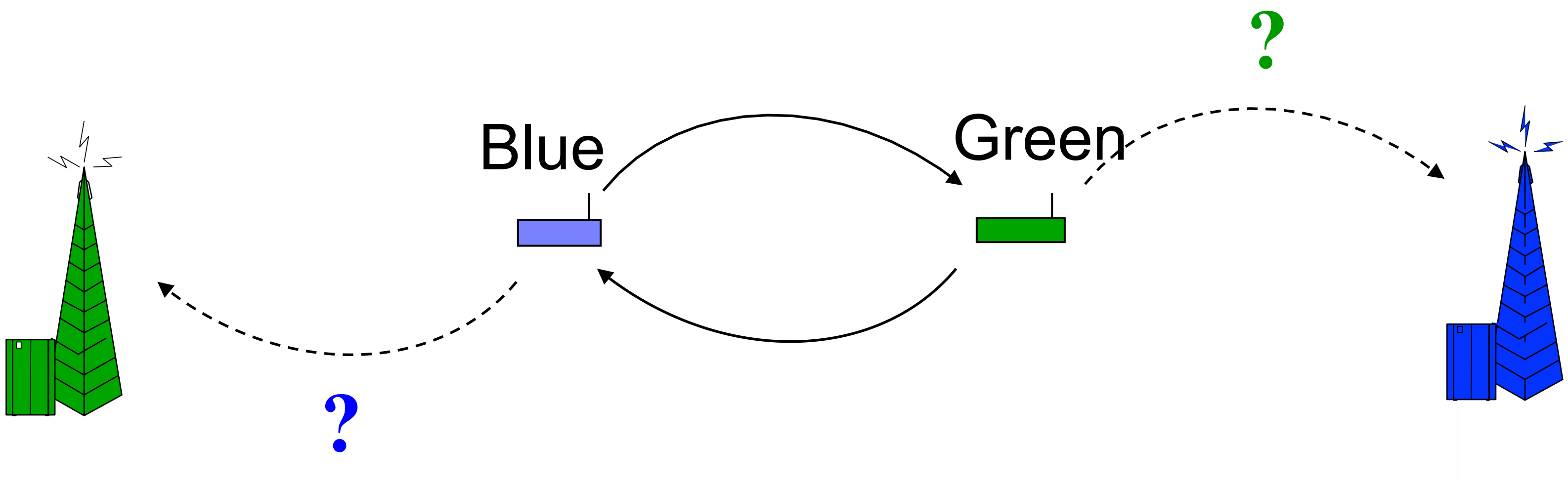
Evolutionary Game Theory
A Simple Model

- Assume a large population of players with hardwired strategies
- We suppose the entire population play strategy s
- We then assume a ***mutation*** happens, and a small group of individuals start playing strategy s'
- **The question we will ask is whether the mutants will survive and grow or if they will eventually die out**

Evolutionary Game Theory
A Simple Model

- Study the existence of Evolutionarily Stable (ES) strategies
- Note:
  - With our assumptions we start with a **large fraction** of players adopting strategy *s* and a **small portion** using strategy *s'*
  - In random matching, the probability for a player using ***s*** **to meet another player using *s*** is **high**, whereas **meeting a player using *s'* is low**

The Forwarder's Dilemma: A Practical Example



Forwarder Game

- Reward for packet reaching the destination: 1
- Cost of packet forwarding: c ($0 < c << 1$)

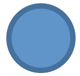
Blue \ Green	Forward	Drop
Forward	(1-c, 1-c)	(-c, 1)
Drop	(1, -c)	(0, 0)

- Have you already seen this game?
  - Prisoners' Dilemma

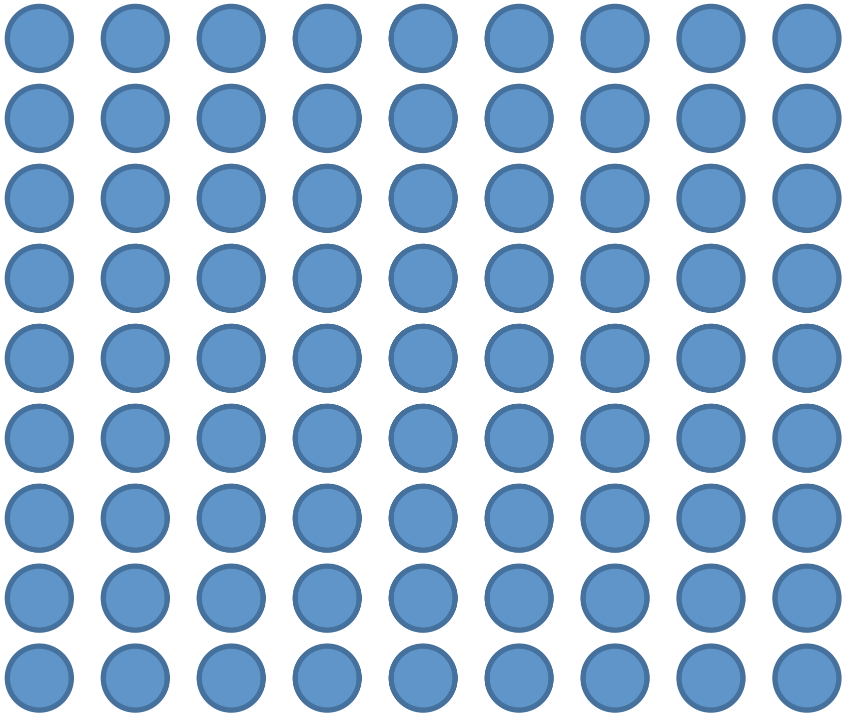
Bio Examples:

1. Lions hunting in a cooperative group
2. Ants defending the nest in a cooperative group

Forwarding Game: A Population



Player strategy hardwired → C



"Spatial Game"

All players are cooperative (Forward) and get a payoff of 1-c

What happens with a mutation?

A Modified Version

		Player 2	
		Cooperate	Defect
Player 1	C	2,2	0,3
	D	3,0	1,1
		1-ε	ε

- Question: ***is "Cooperation" evolutionarily stable***?

Let's Play This Game!

To Cooperate or Not?

Let's have a look to Wild Nature!

Slides are derived from Prof. Hubaux's keynote speech at GameSec 2010:
http://www.gamesec-conf.org/2010/

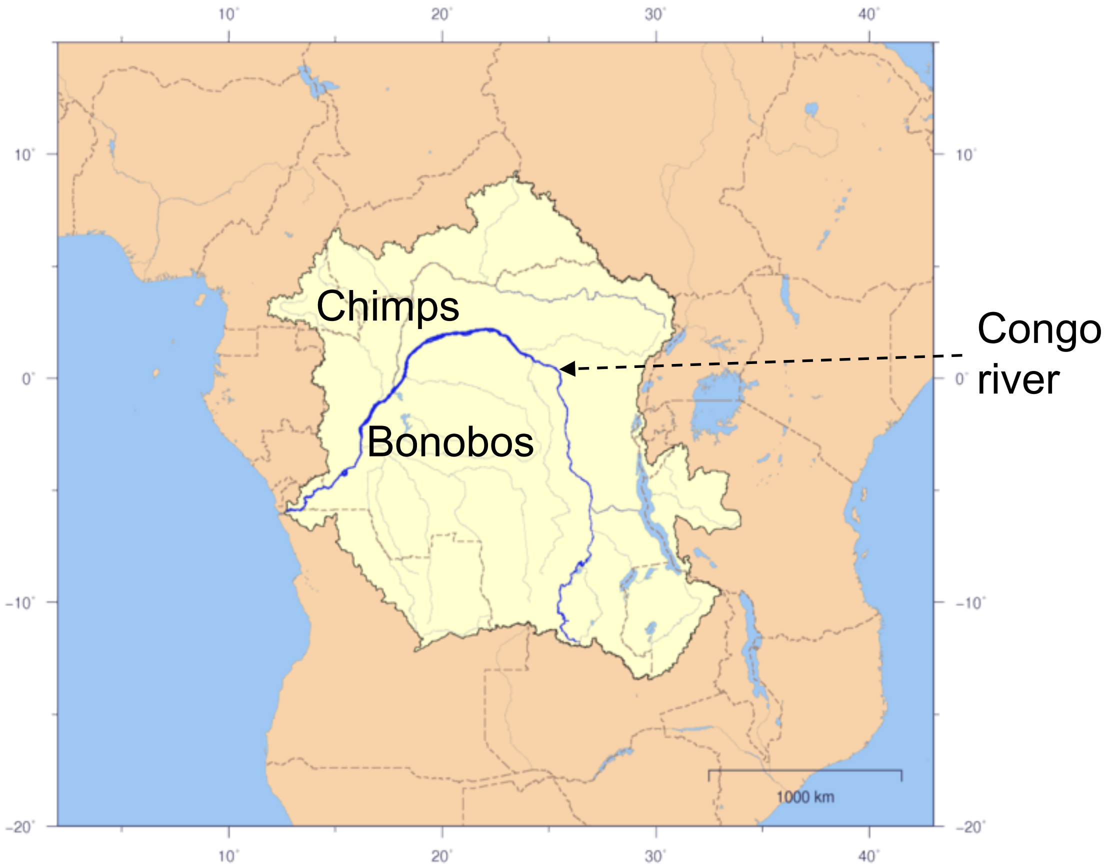
(Non)-Cooperative behavior in wireless networks: Bonobos vs Chimps

Chimpanzee
www.ncbi.nlm.nih.gov

Bonobo
www.bio.davidson.edu

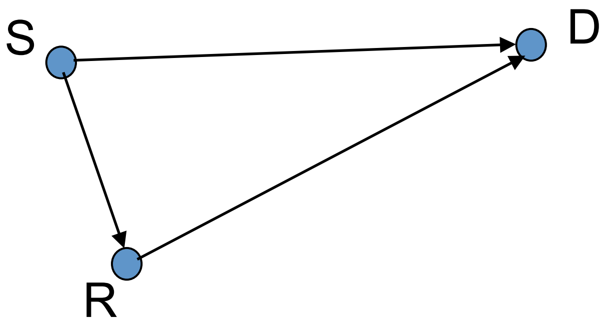
Living places (very simplified)

Cross-layer design…

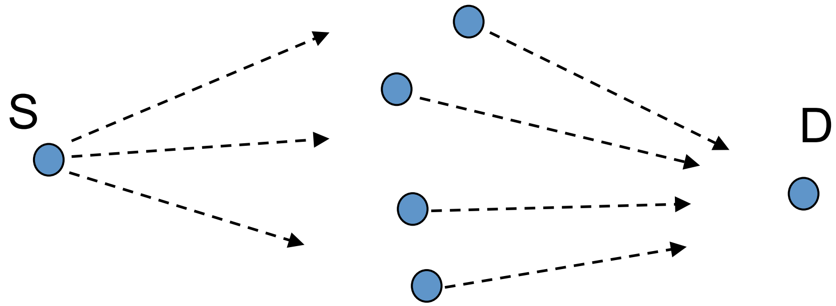


Upper layers (MAC and above)	Non-Cooperative (or “selfish”)
Physical layer	Cooperative

Cooperation between wireless devices (at the physical layer)

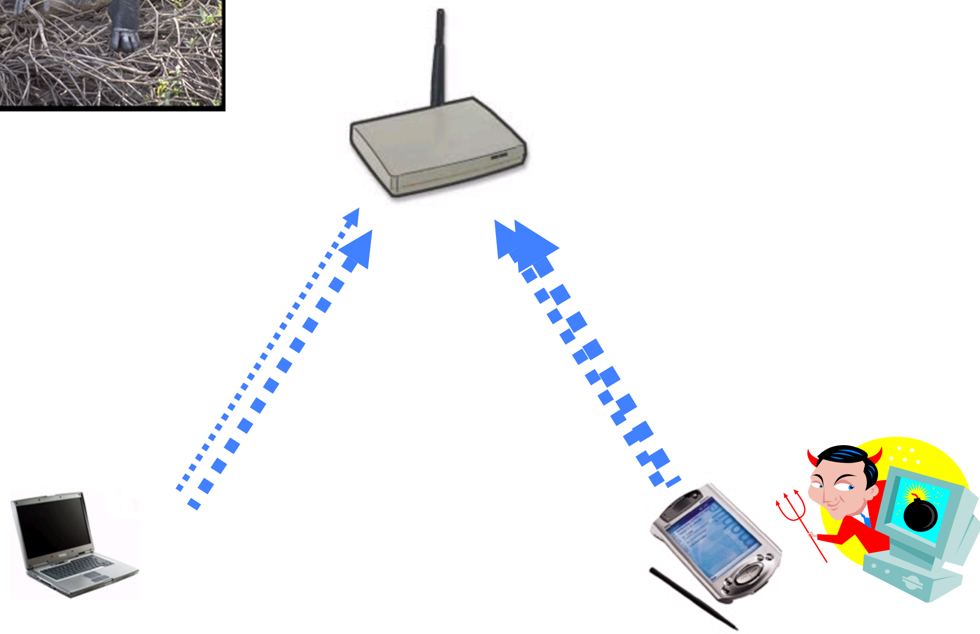


Cooperative relaying



Cooperative beamforming

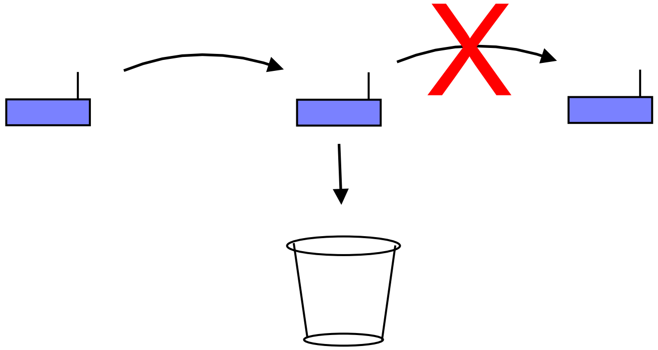
Non-cooperation between wireless devices (MAC and network layer)



Well-behaved node

Well-behaved node / Cheater

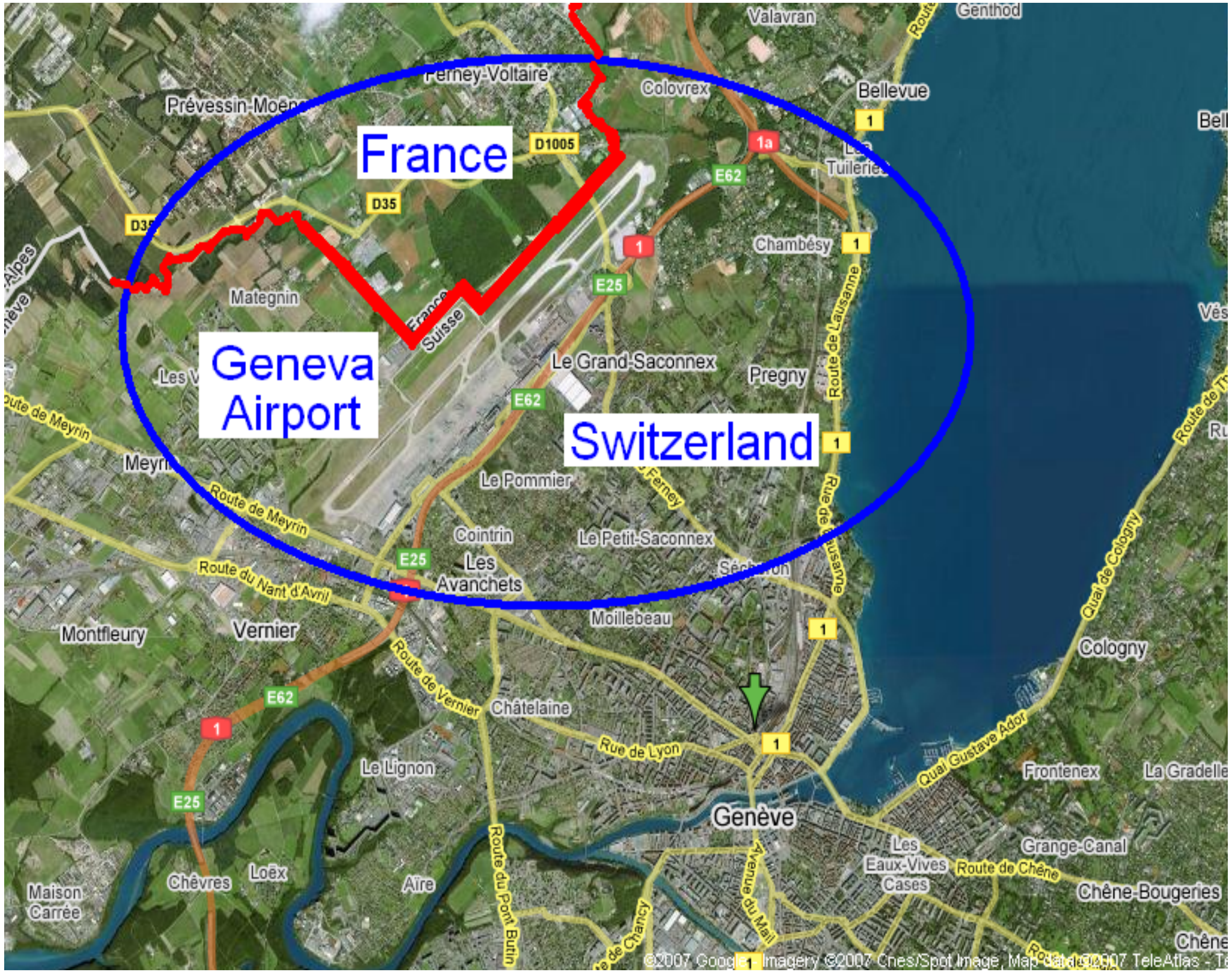
At the MAC layer



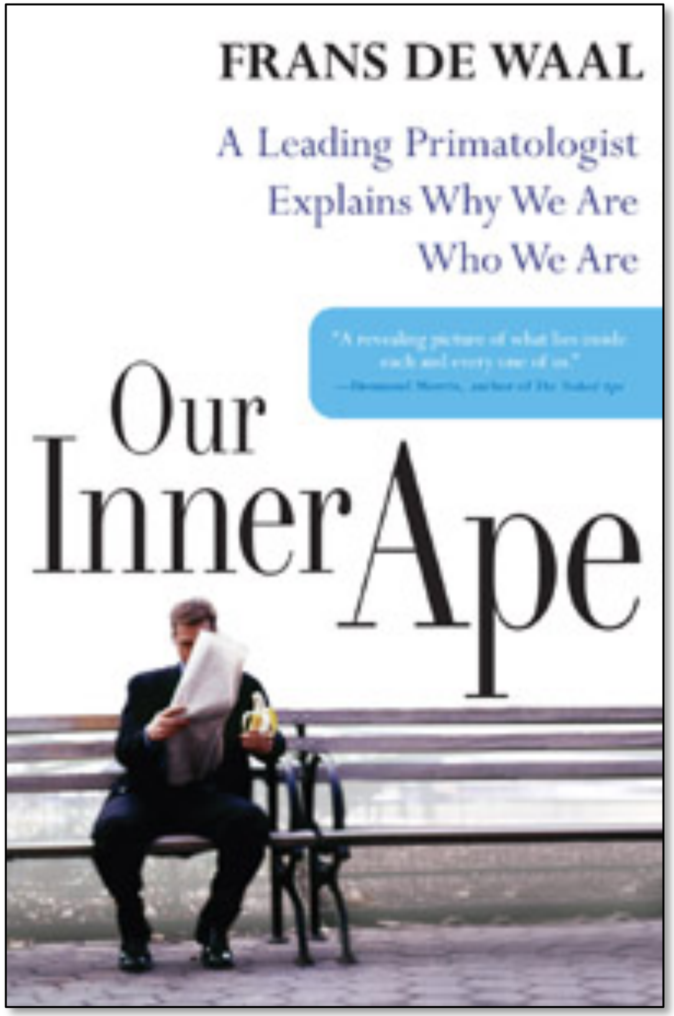
At the network layer

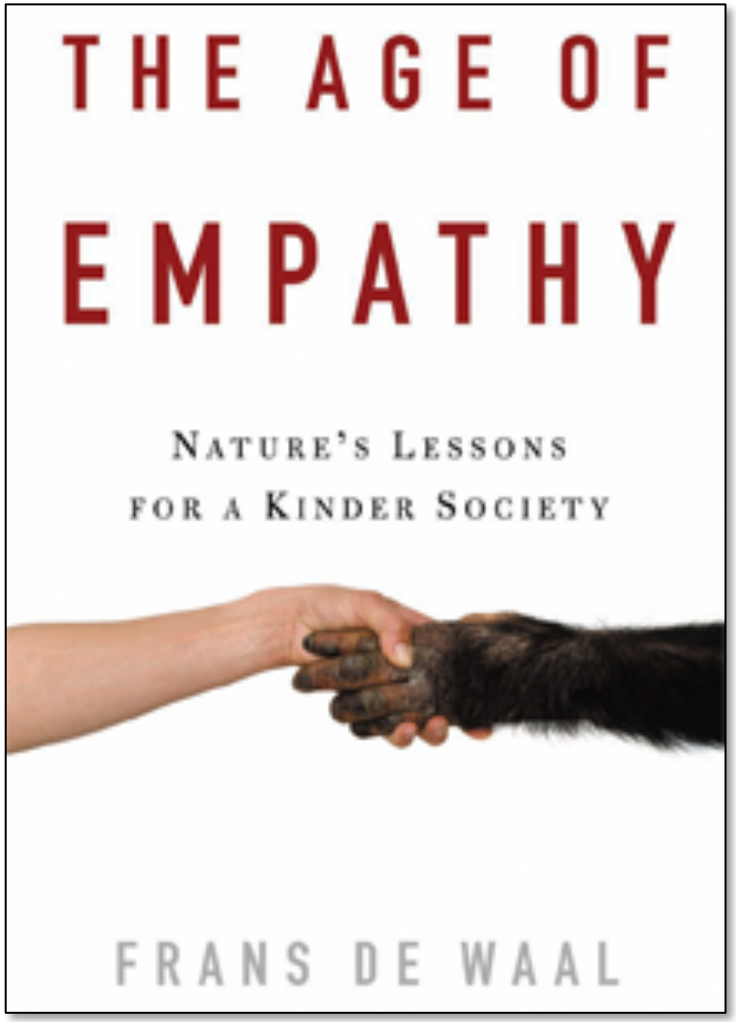
Note: sometimes non-cooperation is assumed at the physical layer; likewise, cooperation is sometimes assumed at the upper layers

(Non-)cooperation between wireless networks: cellular operators in shared spectrum

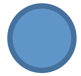


More on Primatology

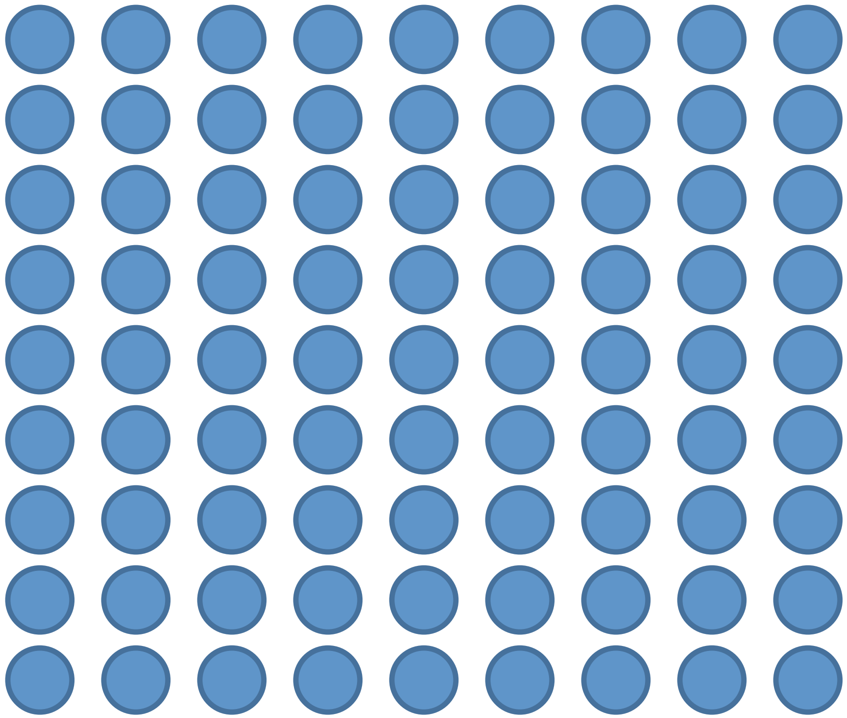




Example



Player strategy hardwired ➔ C

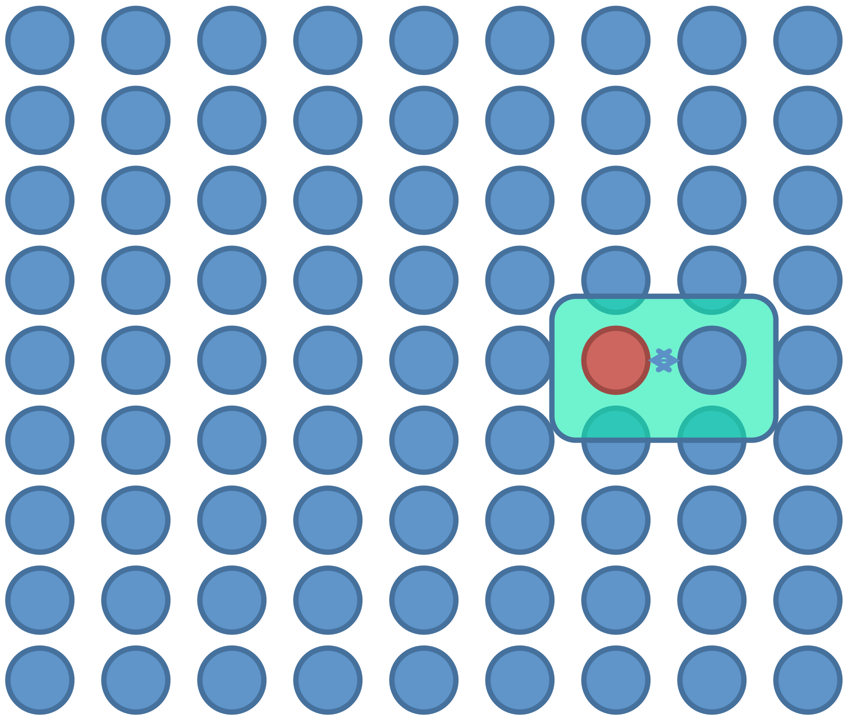


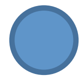
"Spatial Game"

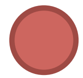
All players are cooperative and get a payoff of 2

What happens with a mutation?

Example



 Player strategy hardwired ➔ C

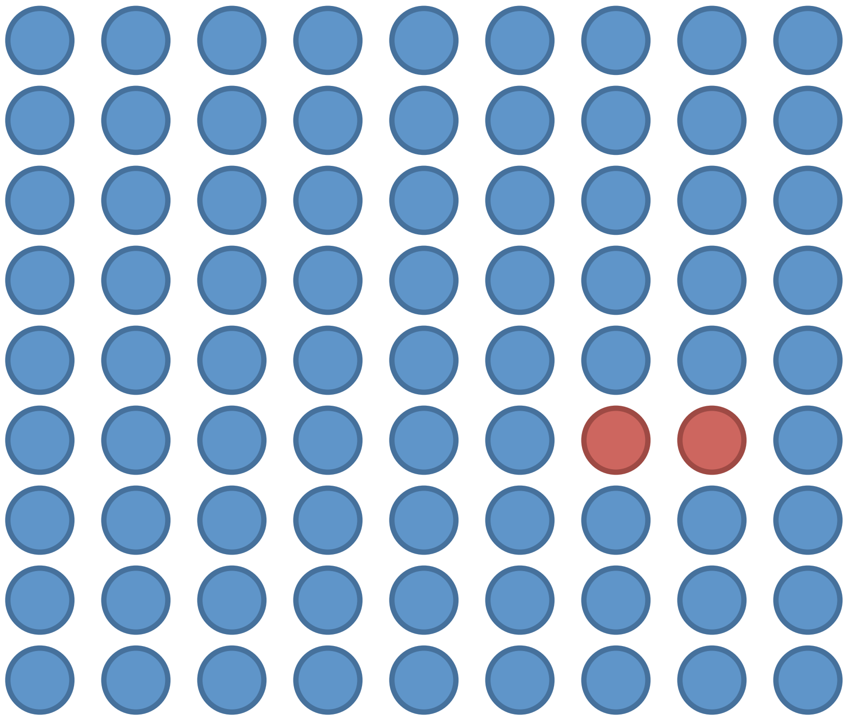
 Player strategy hardwired ➔ D

Focus your attention on this random "tournament":

• Cooperating player will obtain a payoff of 0
• Defecting player will obtain a payoff of 3

Survival of the fittest:
D wins over C

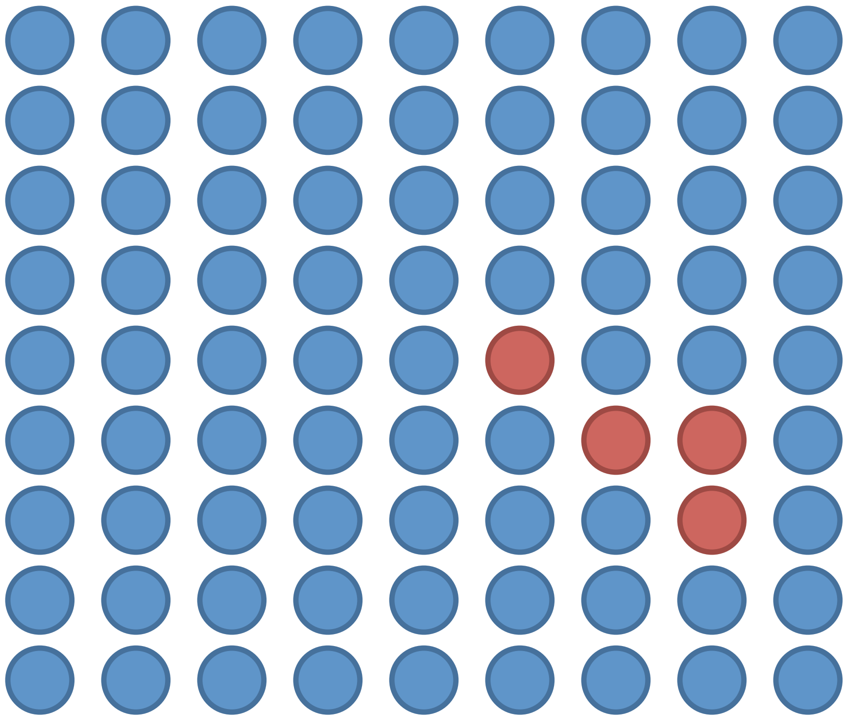
Example



Player strategy hardwired → C

Player strategy hardwired → D

Example



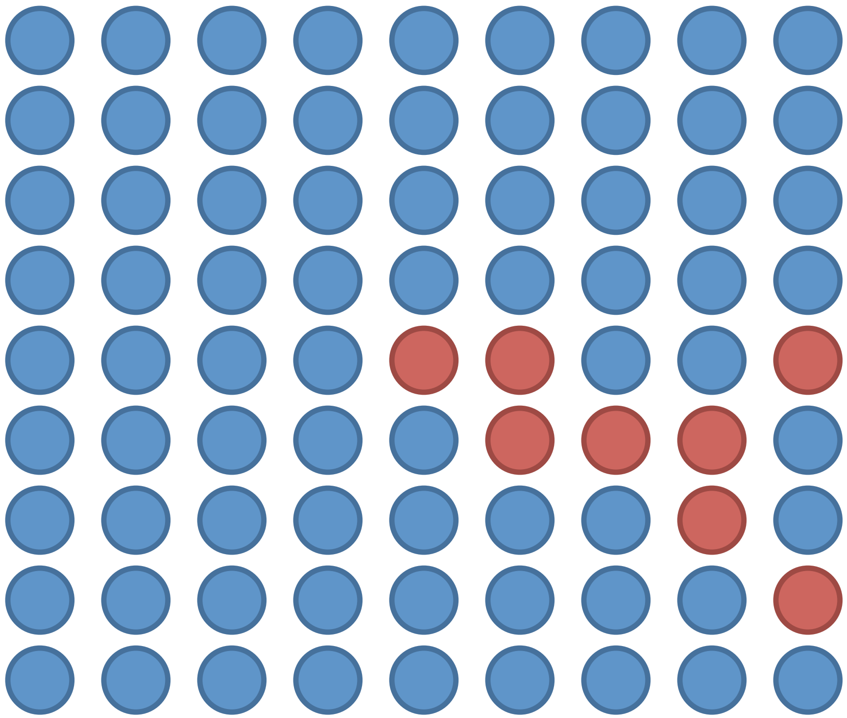
Player strategy hardwired → C

Player strategy hardwired → D

Example

Player strategy hardwired → C

Player strategy hardwired → D



A small initial mutation is rapidly expanding instead of dying out

Let's now try to be a little bit more formal

Is Cooperation ES?

		Player 2 Cooperate	Player 2 Defect
Player 1	C	2,2	0,3
	D	3,0	1,1
		1- ε	ε

For C being a majority

C vs. $[(1-\varepsilon)C + \varepsilon D] \rightarrow (1-\varepsilon)2 + \varepsilon 0 = 2(1-\varepsilon)$

D vs. $[(1-\varepsilon)C + \varepsilon D] \rightarrow (1-\varepsilon)3 + \varepsilon 1 = 3(1-\varepsilon) + \varepsilon$

$3(1-\varepsilon) + \varepsilon > 2(1-\varepsilon)$

➔ C is not ES because the average payoff to C is lower than the average payoff to D

Is Defection ES?

		Player 2 Cooperate	Player 2 Defect
Player 1	C	2,2	0,3
	D	3,0	1,1
		ε	1-ε

For D being a majority

D vs. [(1- ε)D + ε C] → (1- ε)1 + ε 3 = (1- ε)+3 ε

C vs. [(1- ε)D + ε C] → (1- ε)0 + ε 2 = 2 ε

(1- ε)+3 ε > 2 ε

→ D is ES: any mutation from D gets wiped out!

Observations

- ***Lesson 1:*** Nature (Bad Protocols) can suck
  - It looks like animals don't cooperate (Ants and Lions), but we've seen so many documentaries showing the opposite! Why?
  - Sexual reproduction, and gene redistribution might help here

- ***Lesson 2***: If a strategy is strictly dominated then it is not Evolutionarily Stable
  - The strictly dominant strategy will be a successful mutation

Another Game: 3-Strategy

	a	b	c
a	2,2	0,0	0,0
b	0,0	0,0	1,1
c	0,0	1,1	0,0

- 2-player symmetric game with 3 strategies
- Is "c" ES?

c vs. $[(1-\varepsilon)c + \varepsilon b] \rightarrow (1-\varepsilon)\, 0 + \varepsilon\ 1 = \varepsilon$

b vs. $[(1-\varepsilon)c + \varepsilon b] \rightarrow (1-\varepsilon)\, 1 + \varepsilon\ 0 = 1-\varepsilon$

$1-\varepsilon > \varepsilon$

➔ "c" is not evolutionary stable, as "b" can invade it

Is c(or b) ES?

	a	b	c
a	2,2	0,0	0,0
b	0,0	0,0	1,1
c	0,0	1,1	0,0

- So “c” is not ES, as “b” can invade

- NOTE: “b”, the invader, is itself not ES!!
  - But it still avoids dying out completely

NE vs ES

	a	b	c
a	2,2	0,0	0,0
b	0,0	0,0	1,1
c	0,0	1,1	0,0

- Is (c,c) a NE?
- No, because “b” is a profitable deviation

Observations

- ***Lesson 3***:

  If *s* is **not Nash** (i.e., *(s,s)* is not a NE),
  then *s* is **not evolutionary stable** (ES)

If *s* **is ES**, then *(s,s)* **is a NE**

- **Question: is the opposite true?**

Yet Another Game (with 2 NE)

Player 2

		a	b
Player 1	a	1,1	0,0
	b	0,0	0,0
		ε	$1-\varepsilon$

- What are the NE of this game?
  - NE = *(a,a)* and *(b,b)*
- Is *b* ES?

b → 0

a → $(1-\varepsilon)\,0 + \varepsilon\ 1 = \varepsilon$

$\varepsilon > 0$

➔ *(b,b)* is a NE, but it is not ES!

Why NE but not ES?

		Player 2	
		a	b
Player 1	a	1,1	0,0
	b	0,0	0,0
		ε	1- ε

- Why is "b" not ES despite it is a NE?
- This relates to the idea of a weak NE

➔ If *(s,s)* is a ***strict NE*** then *s* is ES

Definition 1: [Maynard Smith 1972]

Bio

In a symmetric 2 player game, the pure strategy $\hat{s}$ is ES (in **pure** strategies) if there exists an $\varepsilon_0 > 0$ such as:

$$\underbrace{(1-\varepsilon)\left[u(\hat{s},\hat{s})\right]+\varepsilon\left[u(\hat{s},s')\right]}_{\text{Payoff to ES } \hat{s}} > \underbrace{(1-\varepsilon)\left[u(s',\hat{s})\right]+\varepsilon\left[u(s',s')\right]}_{\text{Payoff to mutant } s'}$$

for all possible deviations s' and for all mutation sizes $\varepsilon < \varepsilon_0$

Definition 2

ECO or ENG

- In a symmetric 2 player game, the pure strategy $\hat{s}$ is ES (in **pure** strategies) if:

A) $(\hat{s},\hat{s})$ is a symmetric Nash Equilibrium

$$u(\hat{s},\hat{s}) \geq u(s',\hat{s}) \quad \forall s'$$

and

B) if $u(\hat{s},\hat{s}) = u(s',\hat{s})$ then

$$u(\hat{s},s') > u(s',s')$$

Theorem

Definition 1 $\Leftrightarrow$ Definition 2

- Let's see Def. 2 $\Longrightarrow$ Def. 1

Sketch of proof:

- Fix a $\hat{s}$ and suppose $(\hat{s},\hat{s})$ is NE, that is

$$u(\hat{s},\hat{s}) \geq u(s',\hat{s}) \quad \forall s'$$

- There are two possibilities

Theorem (Sketch of Proof)

- **Case 1:**

  $$u(\hat{s},\hat{s}) > u(\hat{s},s') \quad \forall s'$$

  the mutant dies out because she meets $\hat{s}$ often
- **Case 2:**

  $$u(\hat{s},\hat{s}) = u(\hat{s},s') \quad \forall s' \quad \text{but}$$

  $$u(\hat{s},s') > u(s',s')$$

  the mutant does "ok" against $\hat{s}$ (the mass) but badly against s' (itself)

Conclusion

- We've seen a definition that connects Evolutionary Stability to Nash Equilibrium
- Basically, all we need to do is:
  - First check if $(\hat{s},\hat{s})$ is a ***symmetric*** Nash Equilibrium
  - If it is a ***strict*** NE, we're done
  - Otherwise, we need to compare how $\hat{s}$ performs against a mutation, and how a mutation performs against a mutation
  - If $\hat{s}$ performs better, then we're done

Another Game

		Player 2	
		a	b
Player 1	a	1,1	1,1
	b	1,1	0,0
		1- ε	ε

- What is the NE of this game?
  - NE = *(a,a)*
- Is it symmetric? Easy to check

➔a is a good candidate to be ESS

- Is (a,a) a strict NE?

Conditions to be ES

		Player 2	
		a	b
Player 1	a	1,1	1,1
	b	1,1	0,0
		1- ε	ε

- No, it's not a strict NE
  – If you deviate to b, it's easy to notice that *u(a,a)*=*u(b,a)*
- Last Condition
  – How does *u(a,b)* compare to *u(b,b)*?
  – U(a,b) = 1 > u(b,b) = 0
  – It's bigger! We're done: *a* is an ESS

Evolution of a Social Convention

- Evolution is often applied to social sciences
- Let's have a look at how driving to the left or right hand side of the road might evolve

Player 2

Player 1		**L**	**R**
	L	2,2	0,0
	R	0,0	1,1

- Any clues on the interpretation of this game?

Evolution of a Social Convention

	L	R
L	2,2	0,0
R	0,0	1,1

- What's liable to be evolutionary stable in this setting?
- Well, let's find the NE of this game:
  - NE = *(L,L)* and *(R,R)* , which are in fact ***symmetric***
- Are those NE ***strict***?

Evolution of Social Convention

	L	R
L	2,2	0,0
R	0,0	1,1

- Yes, they are strict! We're done:
  - *"L"* and *"R"* are ***both*** ESS

- Lesson 1: We can have multiple ES conventions

Evolution of Social Convention

	L	R
L	2,2	0,0
R	0,0	1,1

- Lesson 2: Multiple ESS ***need not to be equally good***

- This should remind you something we've already seen
  - These are ***coordination games***

The Game of Chicken

	a	b
a	0,0	2,1
b	1,2	0,0

- This is just a symmetric version of the Battle of the Sexes game we've studied extensively

- Biology interpretation:
  - "a" : Individuals that are aggressive
  - "b" : Individuals that are non-aggressive

The Game of Chicken

	a	b
a	0,0	2,1
b	1,2	0,0

- What's evolutionary stable in this game?
- Easy: look for Nash equilibria
  - We know already a lot about this game, let's go straight to the point
- There are 2 NE in pure strategies: *(a,b)* and *(b,a)*

The Game of Chicken

	a	b
a	0,0	2,1
b	1,2	0,0

- Are the pure strategies NE ***symmetric***?
- No, and that's the problem: according to our definition of ESS, neither the pure strategy "a" not "b" can be ES
  - If you had only aggressive genes, they'd do very badly against each other, hence they could be invaded by a gentle gene
  - Of course, vice-versa is also true

The Game of Chicken

	a	b
a	0,0	2,1
b	1,2	0,0

- What should we do? Look at mixed strategies!
- What's the mixed strategy NE of this game?
  - Mixed strategy NE = [(2/3, 1/3) , (2/3 , 1/3)]
  - Note: now it's ***symmetric***
- There is an equilibrium in which 2/3 of the genes are aggressive and 1/3 are non-aggressive

New Definition

- In a symmetric 2 player game, the mixed strategy $\hat{p}$ is ES (in mixed strategies) if:

A) $(\hat{p}, \hat{p})$ is a symmetric Nash Equilibrium

$$u(\hat{p}, \hat{p}) \geq u(p', \hat{p}) \quad \forall p'$$

and

B) if $u(\hat{p}, \hat{p}) = u(p', \hat{p})$ then

$$u(\hat{p}, p') > u(p', p')$$

The Game of Chicken

	a	b
a	0,0	2,1
b	1,2	0,0

- ***<u>Question: can a mixed strategy NE be strict?</u>***
- No, by definition of a mixed NE: payoffs are equal for both pure strategies
- In our example, we need to check (for all possible mixed deviation)

$$u(\hat{p}, p') > u(p', p') \;\; \forall p'$$

The Game of Chicken

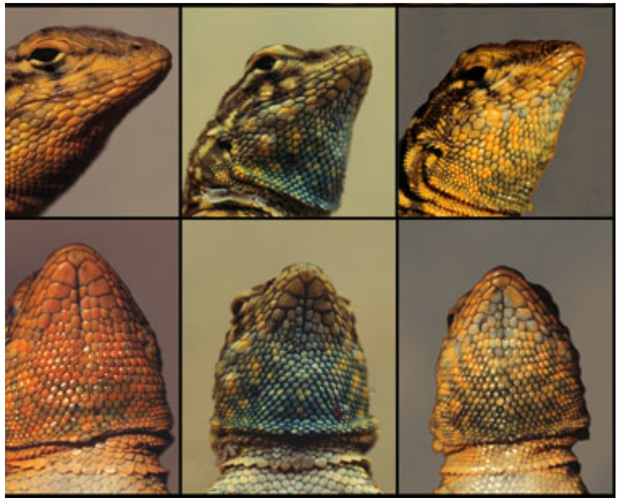
	a	b
a	0,0	2,1
b	1,2	0,0

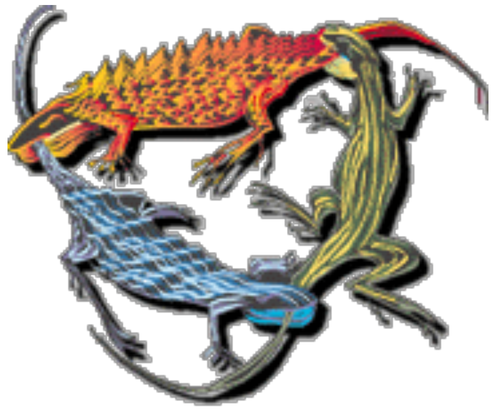
- Instead of a formal proof, let's discuss an heuristic to check that this is true
  - We've got a population in which 2/3 are aggressive and 1/3 are passive
  - Suppose there is a mutation *p'* that is more aggressive than p (e.g. 90% aggressive, 10% passive)
  - Since the aggressive mutation is doing very badly against herself, it would eventually die out
  - Indeed, the mutation would obtain a payoff of 0

Interpretation of Mixed in ES

- ***But what does it mean to have a mix in nature?***
  - It could mean that the gene itself is randomizing, which is plausible
  - It could be that there are actually two types surviving in the population, and this is connected to our alternative interpretation of mixed strategies

Male Reproductive Strategies: The Side-Blotched Lizard





http://bio.research.ucsc.edu/~barrylab/classes/animal_behavior/MALESS.HTM

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2

- We're now going to look at a more general game of aggression vs. non-aggression
- Note: we're still in the context of ***within species competition***
  - So it's not a battle against two different animals, hawks and doves

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2

- The idea is that there is a potential battle against an aggressive vs. a non-aggressive animal
- The prize is food, and it's value is $v > 0$
- There's a cost for fighting, which is $c > 0$

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2

- We're going to analyze ES strategies (ESS)
- We're going to be able to understand what happens to the ESS mix as we change the values of prize and costs

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2

- Can we have an ES population of doves?
- Is *(D,D)* a NE?
  - No, hence "D" is not ESS
  - Indeed, a mutation of hawks against doves would be profitable in that it would obtain a payoff of v

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2

- Can we have an ES population of Hawks?
- Is *(H,H)* a NE?
- It is a symmetric NE if $(v-c)/2 \geq 0$

- **Case 1:** $v>c$ ➔ *(H,H)* is a ***strict*** NE ➔ "*H*" is ESS

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2

- **Case 2:** v=c ➔ (v-c)/2 = 0 ➔ *u(H,H)* = *u(D,H)*
  - Need to check how H performs against a mutation of D
  - Is *u(H,D)* = *v* larger than *u(D,D)* = *v/2?*

➔ H is ESS if $v \geq c$

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2
	$\hat{p}$	$1-\hat{p}$

- What if $c > v$?
  - We know "H" is not ESS and "D" is not ESS
  - What about a mixed strategy?
- **Step 1**: we need to find a symmetric mixed NE

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2
	$\hat{p}$	$1-\hat{p}$

$$\left.\begin{aligned} u(H,\hat{p}) &= \hat{p}\left(\frac{v-c}{2}\right) + (1-\hat{p})v \\ u(D,\hat{p}) &= \hat{p}0 + (1-\hat{p})\frac{v}{2} \end{aligned}\right\} \Rightarrow \hat{p} = \frac{v}{c}$$

$$\Rightarrow \left(\frac{v}{c}, 1-\frac{v}{c}\right)$$

The Hawks and Dove Game

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2
	$\hat{p}$	$1-\hat{p}$

- The mixed NE is not strict by definition
- We need to check:

$$u(\hat{p}, p') > u(p', p') \;\; \forall p'$$

- No formal proof, same heuristic as before

Conclusions from H&D

- In case $v < c$ we have an evolutionarily stable state in which we have v/c hawks
  1. As $v \nearrow$ we will have more hawks in ESS
  2. As $c \nearrow$ we will have more doves in ESS

- What are the payoffs?

Conclusions from H&D

	H	D
H	(v-c)/2, (v-c)/2	v,0
D	0, v	v/2, v/2
	$\hat{p}$	$1-\hat{p}$

- Let's take the D perspective

$$E\left[u(D,\hat{p})\right] = E\left[u(H,\hat{p})\right] = 0\frac{v}{c} + \left(1-\frac{v}{c}\right)\frac{v}{2}$$

- What happens if the cost of fighting grows?

Conclusions from H&D

- The theory we've learned today is amenable to ***identification***
  – We can run experiments and ***measure*** the proportion of H and D
  – From observations, we can deduce the actual values of v/c
- It turns out that this theory is also able to predict outcomes that are not well-known facts