

Foundations of Game Theory for Electrical and Computer Engineering

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- Analyze through the eyes of Game Theory
- Reviewing the basic economic concepts
- Antoine Augustin Cournot (1801-1877)
 French philosopher and mathematician

- Why do we study it? [Game theory answer]
- So far we've seen two types of games
 - Those with few players and few (discrete) strategies
 - Those with a lot of players (e.g. the number game) and few strategies
- CD is a game with few players but a continuum of strategies

- Why do we study it? [Economics answer]
- This game lies between two extreme cases in economics, in situations where firms (e.g. two companies) are competing on the same market
 - Perfect competition
 - Monopoly
- We're interested to understand what happens in the middle

- Given a Cournot Duopoly model of a market, we want to understand what will happen in the market
- We want to understand, from the welfare point of view, if what happens is good or bad for producers/consumers
- There are many applied examples in the Economics of Information Technology (e.g., EC, WEIS, and NetEcon conferences)

- The players: 2 Firms, e.g. Coke and Pepsi
- Strategies: quantities players produce of <u>identical</u> products: q_i , q_{-i}
 - Products are perfect substitutes

• **Cost of production:** c×q

- Simple model of <u>constant marginal cost</u>
- Marginal Cost:
 - Marginal cost is the change in total cost that arises when the quantity produced changes by one unit.

• **Prices:** $p = a - b (q_1 + q_2)$

Price in the Cournot Duopoly Game



• The payoffs: firms aim to *maximize profit*

$$\mathbf{u}_1(\mathbf{q}_1,\mathbf{q}_2) = \mathbf{p} \times \mathbf{q}_1 - \mathbf{c} \times \mathbf{q}_1$$

- Profits = Revenues (Benefit) Costs
- Game vs. maximization problem

$$u_1(q_1,q_2) = p \times q_1 - c \times q_1$$
$$p = a - b (q_1 + q_2)$$

$$\Rightarrow u_1(q_1,q_2) = a \times q_1 - b \times q_1^2 - b \times q_1 q_2 - c \times q_1$$

- Now, we've defined the players, the strategies and the payoffs
- We want to find the NE of this game
- How do we do this?

Cournot Duopoly: Best Response

- First order condition
- Second order condition

 $\frac{\partial u_1(q_1, q_2)}{\partial q_1} = 0$ \mathbf{a}

$$\frac{\partial^2 u_1(q_1, q_2)}{\partial^2 q_1} < 0$$

Cournot Duopoly: Best Response

- First order condition $a 2bq_1 bq_2 c = 0$
- Second order condition -2b < 0[make sure it's a max]

Cournot Duopoly: NE

- We could just find the NE now, right?
- How would you go for doing this?
- Instead, let's see things graphically



BR for Firm I when q_2 = 0

• What would be the BR for Firm 1 if Firm 2 didn't produce at all?

$$\hat{q}_1 = BR_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$$
$$\hat{q}_M = BR_1(q_2 = 0) = \frac{a-c}{2b}$$

• Let's put this quantity on the plot...



Economics Discussion

- What is this quantity we just found called?
- It is called the *monopoly quantity*
- When Firm 2 does not produce, then Firm 1 is a monopolist on the market
- Let's see economic representation!



When marginal revenue = Marginal cost



 \rightarrow q₂ such as q₁ would be the best response for Firm 1

When BR for Firm 1 is $q_1 = 0$?

• We simply take the BR expression and set it to zero

$$\hat{q}_1 = BR_1(q_2) = \frac{a-c}{2b} - \frac{q_2}{2}$$
$$BR_1(q_2) = 0$$
$$\Rightarrow q_{PC} = \frac{a-c}{b}$$

• Let's put this quantity on the plot...



Economics Interpretation

- What is this quantity we just found called?
- It is called the *perfect competition quantity*
- When Firm 2 produces this quantity, the best response for Firm 1 is not to produce
 - Why?



When demand = Marginal cost



When demand = Marginal cost



The game is symmetric

What is the NE of the Cournot Duopoly?

• Graphically we've seen it, formally we have:

$$BR_1(q_2) = BR_2(q_1) \Rightarrow q_1^* = q_2^*$$
$$\frac{a-c}{2b} - \frac{\hat{q}_2}{2} = \hat{q}_2$$
$$\Rightarrow q_1^* = q_2^* = \frac{a-c}{3b}$$

• We have found the **COURNOT QUANTITY**



Cournot Duopoly: Observations

- This game is different from the games we've seen so far:
 - Partnership game
 - Investment game
- In those games, the more the other player would "do" the more I would do
- → Strategic Complements

Cournot Duopoly: Observations

- In this game, the more the other player do, the less I would do
- → This is a game of **<u>Strategic Substitutes</u>**
 - Note: of course the goods were substitutes
 - We're talking about strategies here

Cournot Duopoly: What about the Market?

- Let's now take the perspective of the market and not of a single player
- What about the total industry profits?
 - Are they maximized?
 - Where on the plot we drew before, are industry profits maximized?



Where else are industry profits maximized?



Cartels, Agreements ...

- How could Firm I and Firm 2 set an agreement so as to profit more from the market?
- E.g.: they could decide both to produce half of the monopoly quantity and they would earn more
 - Can you see this on the previous plots?

Cartels, Agreements ...

- What is wrong with this agreement?
- What is the BR for a player? Can you see on the graph where such an agreement would end up?
- Is there anything else wrong in this reasoning?
 - What happened to the production quantities?
 - The market is not fully exploited
 - So?

Cournot Duopoly: Final Word

 How do quantities and prices we've encountered so far compare?

