Foundations of Game Theory for Electrical and Computer Engineering

Mohammad Hossein Manshaei

manshaei@gmail.com

1393

Contents

- Nash Equilibrium
  - ✓ Definition
  - ✓ Motivation
- Rationality vs NE
- Finding a Nash Equilibrium
- Nash Equilibrium vs Dominance

Introducing Nash Equilibrium

- So in the partnership game we've seen what a NE is…
- Recall the numbers game: what was the NE there?
- Did you play a NE?

➔ Although NE is a central idea in game theory, be aware that **it is not always going to be played**

➔ By **repeating** the numbers game, however, we've seen that **we were converging to the NE**

Definition (1): **Nash Equilibrium**

A strategy profile ($s_1^*, s_2^*, \ldots, s_N^*$) is a **Nash Equilibrium (NE)** if, for each i, her choice s_i^* is a best response to the other players' choices s_{-i}^*

Nash Equilibrium = Mutual best responses

Definition (2): **Nash Equilibrium**

At **Nash Equilibrium** no player can increase its payoff by deviating unilaterally.

Definition (3): **Nash Equilibrium**

Strategy profile s^* constitutes a **Nash Equilibrium** if, for each player *i*,

Where: $u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*), \forall s_i \in S_i$

$u_i \in U$ utility function of player i

$s_i \in S_i$ strategy of player i

Contents

- Nash Equilibrium
  - ✓ Definition
  - ✓ Motivation
- Rationality vs NE
- Finding a Nash Equilibrium
- Nash Equilibrium vs Dominance

Is it useful after all!

WHAT ARE THE MOTIVATIONS FOR STUDYING NE?

Why NE?

- Why is it an important concept?
  - It's in textbooks ☺
  - Nash became famous afterward ☺
  - It's used in many applications
- Don't jump to the conclusion that now we know NE, everything we've done so far is irrelevant

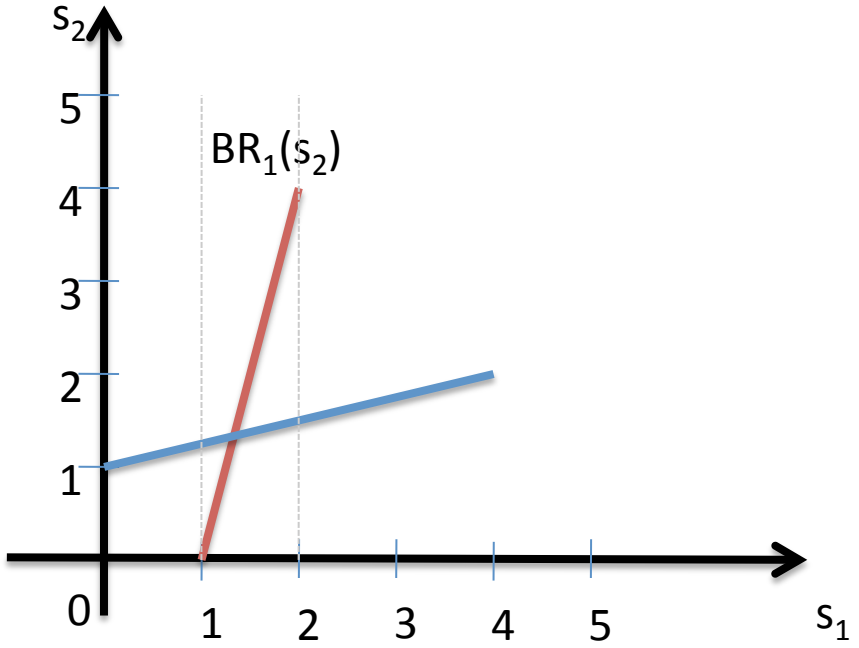
NE: Motivations
No Regret

- Holding everyone else's strategies fixed, no individual has a ***strict*** incentive to move away
- Having played a game, suppose you played a NE: looking back the answer to the question "Do I regret my actions?" would be "No, given what other players did, I did my best"

NE: Motivations
Self-Fulfilling Belief

- If I believe everyone is going to play their parts of a NE, then everyone will in fact play a NE

- Why?



Contents

- Nash Equilibrium
  - ✓ Definition
  - ✓ Motivation
- Rationality vs NE
- Finding a Nash Equilibrium
- Nash Equilibrium vs Dominance

NE: Observations

- It is not always the case that players play a NE!
  - E.g.: in the numbers game, we saw that playing NE is not guaranteed
- Rationality → NE is NOT true!!!

Traveler's Dilemma

Two players attempt to maximize their own payoff, without any concern for the other player's payoff.

Traveler's Dilemma

- *An airline loses two identical suitcases of two travelers*
- *The airline is liable for a maximum of $100 per suitcase*
- *The manager asks travelers to write down the amount of their value at no less than $2 and no larger than $100.*
  1. *If both write down the same number and reimburse both travelers that amount.*
  2. *If one writes down a smaller number than the other, this smaller number will be taken as the true dollar value*
     - *$2 extra will be paid to the traveler who wrote down the lower value*
     - *$2 deduction will be taken from the person who wrote down the higher amount.*
- *What strategy should both travelers follow to decide the value they should write down?*

Contents

- Nash Equilibrium
  - ✓ Definition
  - ✓ Motivation
- Rationality vs NE
- Finding a Nash Equilibrium
- Nash Equilibrium vs Dominance

Finding NE point(s)

- Let's play some very simple games involving few players and few strategies

➔Get familiar with finding NE on normal form games

- We will have a glimpse on algorithmic ways of finding NE and their complexity

Find NE: A Simple Game

		Player 2		
		l	c	r
Player 1	U	0,4	4,0	5,3
	M	4,0	0,4	5,3
	D	3,5	3,5	6,6

- Is there any dominat**ed** strategy for player 1 or 2?

- What is the BR for player 1 if player 2 chooses **left**?
- What is the BR if player 2 chooses **center**?
- What about **right**?

- Can you do it for player 2?

Find NE: A Simple Game

Player 1 \ Player 2	l	c	r
U	0,4	4,0	5,3
M	4,0	0,4	5,3
D	3,5	3,5	6,6

- $BR_1(l) = M$ $\quad BR_2(U) = l$
- $BR_1(c) = U$ $\quad BR_2(M) = c$
- $BR_1(r) = D$ $\quad BR_2(D) = r$

What is the NE?
Why?

Find NE: A Simple Game

- It looks like each strategy of player 1 is a BR to something
- And the same is true for player 2
- Deletion of dominated strategies wouldn't lead anywhere here…

- Would it be rational for player 1 to chose "M"?

Another Simple Game

Player 1 \ Player 2	l	c	r
U	0,2	2,3	4,3
M	1,1	3,2	0,0
D	0,3	1,0	8,0

- What is the NE for this game?
- What's tricky in this game?
  - Do BR have to be unique?
- Are players happy about playing the NE?

Contents

- Nash Equilibrium
  - ✓ Definition
  - ✓ Motivation
- Rationality vs NE
- Finding a Nash Equilibrium
- Nash Equilibrium vs Dominance

NE vs. Dominance

- We've seen how to find NE on a normal form game
- We've seen how NE relates to the idea of BR
  - We have a NE when the BR coincide

- What is the relation between NE and the notion of dominance?

NE vs. Dominance

Player 1 \ Player 2	α	β
α	12, 12	19, 8
β	8, 19	14, 14

- What is this game?
- Are there any dominated strategies?
- What is the NE for this game?

NE vs. Dominance

- **Claim:** no strictly dominated strategies could ever be played in NE

- Why?

➔A strictly dominated strategy is never a best response to anything

- What about weakly dominated strategies?

NE vs. Dominance

		Player 2	
		l	r
Player 1	l	1, 1	0, 0
	r	0, 0	0, 0

- Are there any dominated strategies?
- What is the NE for this game?

NE vs. Dominance

- First observation: the game has 2 NE!
- Informally we've seen that a NE can be:
  - Everyone plays a BR
  - None has any strict incentive to deviate

- What's annoying here? What is the prediction game theory leads us to?
- Is that reasonable?

Pareto-Optimality

How to choose between several Nash equilibria ?

Pareto-optimality: A strategy profile is Pareto-optimal if it is not possible to increase the payoff of any player without decreasing the payoff of another player.

Pareto Optimality: Example

Pareto-optimality: It is not possible to increase the payoff of any player without decreasing the payoff of another player.

Blue \ Green	X	Y	V	W
A	(3, 9)	(0, 6)	(0, 4)	(1, 2)
B	(1, 3)	(3, 6)	(6, 1)	* (6, 3)
C	(4, 2)	(4, 1)	(2, 2)	* (8, 2)
D	(3, 7)	* (4, 5)	(2, 6)	* (4, 7)